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Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

CONTENTS

February, 1960

Papers

(Over)

Page

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

HYDROLOGIC STUDIES BY ELECTRONIC COMPUTERS IN TVA

Willard M. Snyder,[1] M. ASCE

SYNOPSIS

Application of electronic computers in TVA watershed studies is presented. A comprehensive multiple regression analysis program for the IBM 704 computer is described. The use of this program in developing and studying hydrologic prediction equations is illustrated by discussion of numeric results of fitting two equations to data.

INTRODUCTION

Hydrologic studies by the TVA are carried on under a program of research and development of watersheds, which are tributary areas of the Tennessee River. The program is administered by an Advisory Committee on Tributary Watersheds. The purpose of the program and the role of hydrologic investigations are contained in the first two statements of Understanding and Beliefs, as formulated by the Advisory Committee. These are as follows:

1. The welfare of the people is the clear goal of the tributary watershed program, and their full participation is essential to its success.

2. Research activity on hydrologic with related problems and developmental watershed activities are broadly distinguishable. Both may be carried on in the same watershed area, but each is designed to achieve different but related ends. Research activities are designed to produce reliable facts from experiments with single or defined combinations of controlled practices. These facts

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Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

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are basic to the guidance of action programs in developmental watershed projects, where practices are not controlled.

TVA has a variety of programs for development of the resources of the watersheds of the Tennessee Valley. These include cooperative research projects with state universities, test demonstration farms, fertilizer research, and reforestation activities. However, these programs, when undertaken separately, do not achieve maximum development of resources. If the welfare of the people be the primary concern, watershed research and development must be undertaken on an integrated basis. The problems for which the best possible solutions are sought are found in the interaction of such factors as land-use, soils-water relationships, and the social and economic levels of the areas.

A watershed approach to an integrated solution is necessary if one wishes to completely evaluate a program of development and improvement. Starting with rainfall as one of the resources of an area, the changing hydrology must be considered in relation to the total development of resources of that area. Therefore, hydrologic studies and analyses are undertaken as one part of a major program. Causes and effects of many varying factors must be evaluated as precisely as possible on watersheds designated for research activities. These results form a basis for development programs on other projects.

Evaluation of hydrologic data in an integrated program requires development of ways and means for (1) the adjustment of data to eliminate variability inherent in uncontrolled experiments, (2) detection of hydrologic changes, and (3) the synthesis of hydrologic response under changing watershed practices. To accomplish these it is necessary to formulate and evaluate effects of varying amounts and patterns of rainfall, soil moisture levels and rates of loss, among others, and eliminate their effect on volume of runoff, peak rates of discharge, and rates of sediment movement so that the residual data, after this hydrologic adjustment, are less variable than before adjustment. These residual data then form the basis for evaluation and synthesis of external effects, such as watershed management, vegetal cover, and possibly soils types.

Much of the analytic effort involved in hydrologic studies is, of course, reducible to established computational forms. Where numeric procedures exist or can be devised they can usually be programmed for electronic computation. From a practical standpoint, however, only those procedures should be programmed in which the volume of repetitive work justifies the initial cost of program preparation.

TVA COMPUTERS

Electronic computation in hydrologic studies in TVA began in September, 1955 with use of facilities at the National Laboratory at Oak Ridge, Tennessee. In December, 1957 the TVA Division of Water Control Planning obtained a small Royal-McBee LGP-30 computer. In June, 1958 installation of a high speed IBM 704 computer was completed. Hydrologic computations have been programmed for both TVA machines.

Programming for computers in TVA is on a user basis. That is, programs for hydrologic computation are written by engineers engaged in hydrologic studies. Programs dealing with system operations are written by engineers engaged in river control activities. To this end, selected engineers and engineering aides from the Hydraulic Data Branch, and every other using Branch, have been trained in computer programming.

Hydrologic problems programmed for the small LGP-30 computer were those which arose during basic processing of information, such as computation of mean daily discharge from time-gage height abstracts, computation of daily suspended sediment loads from discharge and concentration data, and computation of indices of antecedent precipitation.

Hydrologic problems programmed for the large IBM 704 computer were those which contain complex or voluminous computations. Two programs have been very useful. One computes daily values of a soil moisture index from rainfall, runoff, and evaporation for extended periods of record. The second is a program for comprehensive multiple regression analysis based on the method of least squares.

THE MULTIPLE REGRESSION PROGRAM

The multiple regression program is very definitely the most valuable of the current hydrologic computer programs. The method of least squares as a technique for evaluating the simultaneous effect of many factors has become well established in hydrology. Coupling this technique with regression analysis makes the scope of analytic results almost unlimited, and appears to offer the best approach to hydrologic evaluation which is a necessary part of the TVA Tributary Watershed Studies.

The multiple regression program used in TVA was written to include five major phases of analysis in fitting prediction equations to observational data. These will be described in sequential order of computation.

Product Summation.—Input data for the program consist of individual observations and associated parameters which have been computed for each observation. Generation of a triangular matrix of product-summations is the first phase of computation. This matrix is the set of coefficients of the simultaneous normal equations for least-squares fitting. A 24 x 24 matrix on 500 observations can be generated. The triangular matrix is printed.

Simultaneous Equations.—A symmetrical square matrix is generated from the triangular matrix. This matrix is inverted and the regression coefficients are computed. Both are printed. Following this, various statistical quantities resulting from fitting the equation are computed and printed. These include sums of squares for regression and for residuals, the correlation coefficient, the error of prediction and the error of regression, the standard errors of the regression coefficients, and the arithmetic mean of each variable.

Predicted Values and Prediction Errors.—After the statistical results of fitting are printed the observations are again read into the computer. The predicted value for each observational set is then computed from the regression coefficients and the independent variables. The predicted value is subtracted from the observed value to give the prediction error for each observation. Both predicted value and prediction error are printed.

Predicted Values and Variance of Prediction.—To this point in the regression program the operator has not been able to exercise any options or program alternatives on computation or output. The last two phases of computation are optional.

It is sometimes desirable to test the statistical variability of the regression equation when used as a prediction device. The prediction errors computed for the initial data set do not meet this requirement. However, standard statistical

procedures can be found for this computation.[2] This procedure, an operation on the inverse matrix, was written into the program.

To use this optional portion of the program a list of synthetic observations must be prepared in advance, and punched on cards or placed on magnetic tape along with the list of observations used in fitting. The only requirement in setting up the synthetic list is to set the independent variables at values at which it is desired to test the variability of prediction. The machine computes the predicted value and the variance of the prediction for each synthetic observation in the set. For a simple X-Y correlation this portion of the regression program would compute the familiar hyperbolic confidence limits about the line of regression. For many independent variables the model is complex and the arithmetic becomes quite lengthy. More than 200 product-summations are required. For example, to evaluate the variance for a single prediction in a 10-term equation, and the computer is a tremendous time saver in this operation.

Significance of Variables.—The last phase of the computer program was written to use in testing the different independent variables for statistical significance. This phase of the program also is optional.

It is true that the standard errors of the regression coefficients can be used to form a test of significance on the statistic "t". This test, however, is limited to one variable. When several variables in combination are to be tested for significance, the regression equation must be re-evaluated with this combination of terms left out. The resulting change in the sum of squares absorbed by the regression forms the basis for analysis of variance, using the statistic "F" to determine the significance of the entire combination of variables under consideration.

The TVA multiple regression program allows any number of re-evaluations of the regression equation with any number or any combination of variables left out. The combinations of terms dropped are independent of the order in which the re-evaluations are made. With this complete freedom of choice, the analyst specifies beforehand the various re-evaluations which are desired. For each reduced equation the program causes computation of the reduced inverse matrix and regression coefficients and the other various elements as determined for the first evaluation. The predicted values, however, are not computed. Comparison of the solutions for all the reduced equations, with the initial total equation can then be made on all these elements of the solution. These, of course, are in addition to the conventional analysis of variance.

EQUATION FOR PEAK DISCHARGE

Some examples of the equations being evaluated by the computer program for regression analysis may be of interest. It is, perhaps, a seeming contradiction, that proper utilization of computers requires a better working knowledge of mathematics than when investigations are based purely on empiricisms and short-cut assumptions. However, computer programs must follow precise mathematics if part of the goal of computer utilization is more precise analysis.

One particularly vexing problem in TVA hydrologic studies has been the prediction of peak discharges for some small, single-practice research watersheds in western North Carolina. The watersheds are operated as a coopera-

[2] "STATISTICAL THEORY IN RESEARCH," by R. L. Anderson and T. A. Bancroft, McGraw-Hill Book Co., New ;York, 1952, p. 202.

tive project between TVA and North Carolina State College. Four watersheds, ranging in size from 3.7 acres to 5.6 acres are being operated under a statistically designed experiment. Results from four different covers on the four watersheds will be studied by analysis of variance following a 4 x 4 latin square design. Preliminary to this analysis it will be necessary to establish the relationship of the important hydrologic and meteorological variables, so that data can be adjusted for varying meteorological conditions.

On these small watersheds the normal hydrologic procedure of correlating peak discharge with peak-producing rain, maximum intensity, antecedent moisture, and so one, would not produce workable peak equations. The runoff at the measuring flumes responds so quickly to changes in rainfall intensity that there are quite often several peaks during a storm, each peak representing a burst of rainfall. After several trials at analysis, it was decided that the pattern or time distribution of rainfall would have to be considered. The following development leads to such an equation.

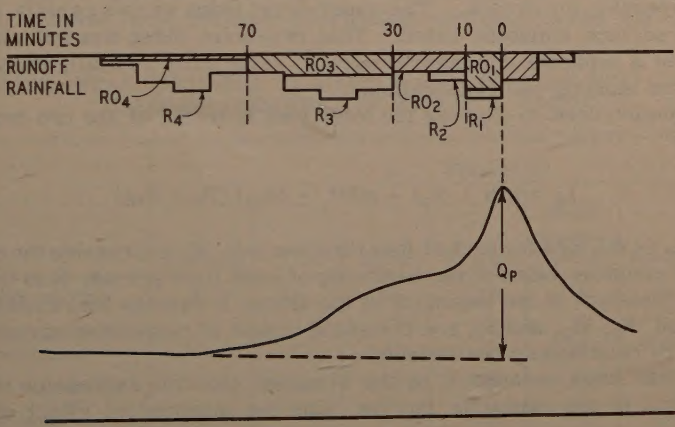

FIG. 1.—DEFINITION OF RUNOFF PERIODS WITH RELATION TO PEAK DISCHARGE

Any ordinate of a storm hydrograph can be predicted from a unit hydrograph and incremental volumes of runoff. Peak discharge, being one ordinate, can be predicted by an equation which expresses the unit hydrograph principle:

$$Q_p = U_1(RO_1) + U_2(RO_2) + U_3(RO_3) + U_4(RO_4) \ldots + U_n(RO_n) \ldots \quad (1)$$

in which Q_p is peak discharge, U represents the set of unit hydrograph coefficients, and RO is the volume of runoff by periods.

The quantities U in Eq. (1) can be considered time-distribution coefficients and they do not have to be at uniform intervals.[3] For the western North Carolina watersheds the coefficients were assigned a time definition as shown in Fig. 1. In this schematic diagram rainfall and the resulting hydrograph are plotted to a common time scale. Zero time is taken as the time of peak, and the time definition of the coefficients extends backward to the beginning of rainfall.

[3] "HYDROGRAPH ANALYSIS BY THE METHOD OF LEAST SQUARES," by W. M. Snyder, ASCE Proceedings Paper No. 793, September, 1955.

The volumes of rainfall, R, can be obtained from a recording rain gage. It is necessary to devise a method to predict runoff from rainfall for each of the four time periods. Runoff is an excess of rainfall and can be computed for each period from rainfall minus loss:

$$RO_t = R_t + L_t, \qquad (2)$$

where loss is considered negative.

Loss was considered in two parts. The first part was an initial amount which varied with season but which was independent of storm rainfall. The second part was a progressive amount which accumulated during the storm at a rate governed by the accumulation of storm rainfall, but also dependent upon initial soil moisture. A soil moisture index, used in this relation, was computed by modifications to a previously reported budgetary system which computes a two-level index.[4] The deep-level index varies slowly, and was used as the expression for season. The upper-level index varies rapidly, and was used as a surface moisture index. This two-level index was calculated for each day by a separate computer program, using daily rainfall, daily runoff, and adjusted monthly pan evaporation.

The equation used to express the total loss in terms of the two component parts was

$$L_t = n(N + N_o) + m(M_t + M_{ot})\,(R_t + R_{ot}) \qquad (3)$$

in which L_t is the loss for each of four time periods, M_t represents the surface-layer soil moisture index at the beginning of each time period, N is the deep-layer soil moisture at the beginning of the storm, R denotes the rainfall during each period, N_o, M_o, and R_o are threshold values of respective variables, and n and m are relationship coefficients.

Eq. 3 was used because it is the simplest possible expression of interaction effect of two causative factors, plus the independent effect of a third factor.[5]

The surface soil moisture index, M, may be considered as increasing directly with rainfall. This is approximately correct when direct runoff is a small portion of the rainfall. From this is derived:

$$\begin{aligned} M_4 &= \text{surface-layer moisture index at beginning of storm} \\ M_3 &= M_4 + R_4 \\ M_2 &= M_4 + R_4 + R_3 \\ M_1 &= M_4 + R_4 + R_3 + R_2 \end{aligned} \qquad (4)$$

Eq. 4 can be used with Eq. 3 to define the loss for each of the four time periods. Only rainfall during the preceding periods, initial moisture index of the surface layer, and the moisture index of the deep layers are required. Combining the resulting equations for period losses with Eq. 2, these expressions for runoff can be substituted into Eq. 1. The resultant prediction equation for peak discharge is algebraically complex in its coefficients, but simple in its

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[4] "COMPUTATION OF EVAPORATION AND EVAPOTRANSPIRATION FROM METEOROLOGICAL OBSERVATIONS," by M. A. Kohler. Paper prepared for presentation at AMS Meeting in Chicago, March 19-21, 1957.

[5] "SOME LINEAR MODELS FOR MULTIPLE REGRESSION ANALYSIS IN HYDROLOGY," by W. M. Snyder. Duplimat Reproduction by Tennessee Valley Authority, Knoxville, Tennessee, May, 1956.

variables. By collecting coefficient terms the equation can be simplified to the following:

$$Q_p = a + b_1 R_1 + b_2 R_2 + b_3 R_3 + b_4 R_4 + c_1 R_1 M_1 + c_2 R_2 M_2 + c_3 R_3 M_3 + c_4 R_4 M_4 + d M_4 + e N \quad . \quad (5)$$

In Eq. 5 the variables R, M, and N to which the peak, Q_p, is related have been previously defined. The coefficients, however, result from algebraic simplification and have no real hydrologic interpretation. The numerical value of the coefficients can be determined by the method of least squares. The results obtained by thus fitting the equation to data can be analyzed by multiple regression analysis.

Eq. 5 has been evaluated for several sets of watershed cover data from the western North Carolina project. Initial results for this interim stage of the project appear entirely satisfactory. Some numerical results are shown in Table 1. In this table multiple correlation coefficients and the conventional

TABLE 1.—PEAK DISCHARGE REGRESSION ANALYSES FOR WESTERN NORTH CAROLINA COOPERATIVE RESEARCH PROJECT[a]

	Watershed						
	1			2		5	6
	Pasture	Corn	Wheat	Pasture	Over-grazed Pasture	Pasture	Wheat
Number of storms	97	68	65	172	106	44	62
Multiple correlation coefficient	0.76	0.57	0.42	0.79	0.86	0.90	0.88
Error of regression	0.014	0.256	0.118	0.009	0.018	0.102	0.155
Average peak	0.069	0.742	0.374	0.124	0.166	0.660	1.018

[a]Units for errors and averages are cubic feet per second.

statistical parameter, the error of the regression, are tabulated for comparison with the average peaks which were observed under different covers. The multiple correlation coefficients show that, in general, the equation is making significant adjustments to the data. However, this is a relative measure and not too useful. A more absolute check is the error of regression. Table 1 shows that the errors are small for pasture and over-grazed pasture, but that larger errors are associated with corn and wheat.

Another way of studying the equation is by the magnitude of peaks which are predicted and the variance, or probable variation, of those predictions. Table 2 shows predictions for a storm of high intensity with and without rainfall immediately preceding. The predicted peaks are reasonable for all covers. For both the short duration and long duration storms, the peaks are lowest for pasture, slightly higher for over-grazed pasture. However, predicted peaks are highest for corn for the short duration or summer intense storm, and highest for wheat for the winter intense storm. This follows the known cover cycle of poor cover under wheat during the winter-time, following fall planting and poor cover under corn in spring and early summer following spring planting.

The errors of prediction given in Table 2 are simply the square roots of the respective prediction variances. The probable variations are given in this form so that the units are the same as for the predicted peaks. The errors of prediction show that, in general, the differences between peaks for various cover are real. However, some improvements must still be made. Variances for corn and wheat covers must be reduced. Special attention may also have to be given to Watershed 5. This watershed, under certain storm conditions, has very large amounts of subsurface flow, producing very different peaks than if flow were from surface runoff. As a consequence, peak discharge data from this watershed are more variable than from the other areas, and the average residual errors also tend to be large. In attempting to improve the prediction of peak discharges a study will first be made of the prediction errors for the individual storms. These data are available from the machine printout sheets, and an attempt will be made to associate the errors with cover or watershed condition as it varies with the annual cultivation cycle.

TABLE 2.—PREDICTED PEAK AND ERROR FOR WESTERN NORTH CAROLINA COOPERATIVE RESEARCH PROJECT[a]

	Watershed						
	1			2		5	6
	Pasture	Corn	Wheat	Pasture	Over-grazed Pasture	Pasture	Wheat
Storm Parameters (Inches): $R_1 = 0.50$, $R_2 = 0.20$, $R_3 = 0$, $R_4 = 0$, $M = 0.50$, $N = 14.0$							
Predicted peak	0.40	5.80	1.64	0.37	1.01	0.03	1.22
Standard error of prediction	0.05	1.14	0.68	0.04	0.08	0.55	0.51
Storm Parameters (Inches): $R_1 = 0.50$, $R_2 = 0.20$, $R_3 = 0.50$, $R_4 = 1.00$, $M = 1.00$, $N = 20.0$							
Predicted peak	0.43	6.40	8.08	1.04	2.13	3.50	8.99
Standard error of prediction	0.25	4.66	3.45	0.20	0.30	1.84	4.90

[a]Units are cubic feet per second.

EQUATIONS FOR VOLUME OF RUNOFF

An equation for prediction of volume of storm runoff from the small watersheds in western North Carolina is also under development. This equation is very similar to that used for prediction of peak discharge. The similarity lies in the definition of rainfall quantities by short periods of time. There is a difference, however, in that the volume equation has the most precise time definition at the beginning of the storm where loss rates are high and changing rapidly.

Fig. 2 is a diagrammatic definition of the periods of rainfall used in the volume of runoff equation. As in Fig. 1, rainfall and the resulting hydrograph are plotted to a common time scale. However, zero time in Fig. 2 is taken at the beginning of storm rainfall, and time definition extends forward to the end of rainfall.

The volume of runoff equation, based on the rainfall by periods as in Fig. 2, and on soil moisture is:

$$RO = a + b_1 R_1 + b_2 R_2 + b_3 R_3 + b_4 R_4 + b_5 R_5 + c_1 M + c_2 N \quad (6)$$

in which RO is surface plus subsurface runoff, R_t are rainfall amounts dis-

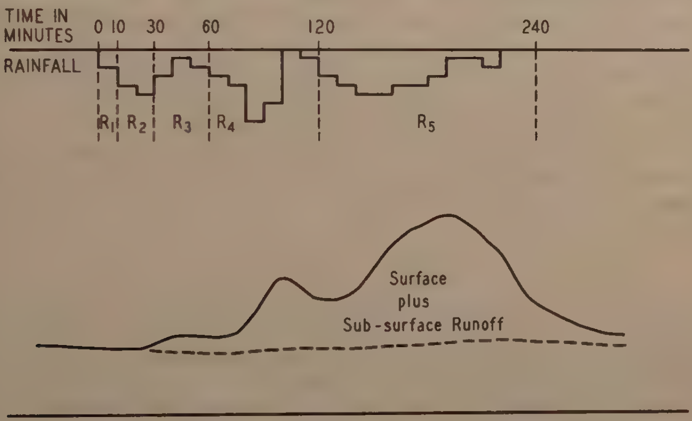

FIG. 2.—DEFINITION OF RUNOFF PERIODS WITH RELATION TO STORM RUNOFF VOLUME

tributed by time periods, and M and N are moisture parameters described previously.

Only a few analyses of this equation can be reported at this time. Results of these fittings are encouraging and are shown in Table 3. Multiple corre-

TABLE 3.—RUNOFF VOLUME REGRESSION ANALYSES FOR WESTERN NORTH CAROLINA COOPERATIVE RESEARCH PROJECT[a]

	Watershed		
	1	5	6
	Pasture	Pasture	Wheat
Number of storms	114	43	46
Multiple correlation coefficient	0.75	0.94	0.90
Error of regression	0.003	0.019	0.025
Average runoff	0.029	0.267	0.161

[a]Units for errors and averages are area inches.

lation coefficients are high and errors of regression are low. Some study of the residual errors of these fittings indicates there may be improvement possible by taking into account the volume of rain above some critical intensity, perhaps for each rainfall period.

COSTS OF COMPUTATION

It would be impractical to evaluate many equations of the distributed rainfall type by desk calculator. The time required for solution of these lengthy equations would allow only a few trials. Many runs of the multiple regression program have been possible on the 704 computer. A cost comparison was made after 17 analyses. Computer costs for these 17 runs totalled $663. This does not include cost of writing the program but does include costs of checking out the program on the machine. It was estimated that the same analyses using desk calculators would have cost about $10,000. This estimate is based on an extrapolation of costs for comparatively short equations. There is no allowance made in this extrapolation for an increasing number of operator errors due to the tedium of lengthy calculations, nor to the possibility of increased personnel turn-over due to pure boredom.

OUTLOOK

The electronic computer will be used at an increasing rate in hydrologic studies. The high speed of computation makes possible the accomplishment of routine work quickly and efficiently. The working time of hydrologists and aides, released from much routine computation, can be used to develop new methods, new techniques, and new machine programs. The low unit cost of analysis when computed electronically allows rapid trial of these new methods over many observations and many data sets.

Rapid strides are being made in development of techniques of data analysis. The great challenge, however, lies in hydrologic synthesis. Hydrologists will not be making full use of computer capabilities until techniques are developed whereby proposed watershed programs can be evaluated by integrating the effects of changes on many small sub-areas. This has been done for water yield, based on infiltration classifications of the sub-areas. But it is important that practical techniques of integration be developed for changes in flood peaks, shape of the hydrograph, and transport of sediment. These are factors which can unquestionably be improved by good land management and should serve as the bases for evaluating proposed programs.

All phases of computer applications, basic processing of observations, analysis of data, and research for new techniques are necessary steps in a continuing program. The end result of this program is to make more and better hydrologic information available to agronomists, foresters, and hydraulic engineers who are working to improve the welfare of the people.

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SCALES OF VISCOUS ANALOGY MODELS FOR GROUND WATER STUDIES

By Jacob Bear[1]

SYNOPSIS

Scales for viscous fluid models used for studying various problems of flow through porous media are described. Consideration is given to some model techniques and limitations. The discussion covers scales for length (including distorted scales), time, discharge, volume, storage coefficient, two liquid flows, and capillary effects.

INTRODUCTION

The movement of fluid through porous media is a complicated phenomenon, and the mathematical equations that describe it can be solved only for a few very simple cases. Difficulties are removed to a large extent when model studies replace the theoretical approach. Among the various types of models which are used for studying ground water movement are the sand model, the electric analogy, the heat analogy, the membrane analogy, and the viscous flow analogy. This paper is concerned with the viscous flow analogy, also known as the Hele-Shaw model.

The viscous flow analogy model consists of two parallel plates mounted together with a capillary interspace (0.5-3.5 mm) between them. The analogy is based on the similitude between the differential equations which describe

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the field of saturated flow of water through porous media and those for laminar flow of a viscous fluid through the capillary interspace between two parallel plates.

The first model of this kind was developed by H. S. Hele-Shaw[2,3] in 1897-1899, for the study of flow patterns around variously shaped bodies. R. Dachler[4] in 1936, was the first to suggest the application of this model to ground water flow studies. Since 1940, the model has been much improved, developed, and intensively used in the Hydrological Laboratory of the Government Institute for Water Supply, The Hague, Netherlands, for studying various problems of ground water movement[5,6,7,8,9]; in the Israel Institute of Technology, Haifa; and by Water Planning for Israel, Tel-Aviv. Other studies and applications have been reported by Todd[10,11,12].

The viscous flow analogy can be used for almost any study of two dimensional flow in porous media whether steady or not. It has been used for seepage through dams and from channels, for tidal fluctuations of the water table, for ground water balances, for studies of the movement of the salt water-fresh water interface in coastal aquifers, and other problems. In 1956, this type of analogy was further improved to include the three dimensional case of horizontal flow in an aquifer with a fluctuating water table.[13,14]

The principles of the viscous flow analogy are well known and described in many articles dealing with this type of model, yet a complete description of the computations of scales for the various cases is missing. This paper is presented as an effort to fill this gap.

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DERIVATION OF MODEL SCALES

The two dimensional flow of ground water with a phreatic surface in a non-isotropic aquifer is described by:

$$K_{xp} \frac{\partial^2 \phi_p}{\partial x_p^2} + K_{zp} \frac{\partial^2 \phi_p}{\partial z_p^2} = S_{op} \frac{\partial \phi_p}{\partial t_p} \quad \ldots\ldots\ldots\ldots \quad (1)$$

For a point on the water table,

$$K_{xp}\left(\frac{\partial \phi_p}{\partial x_p}\right)^2 + K_{zp}\left[\left(\frac{\partial \phi_p}{\partial z_p}\right)^2 - \frac{\partial \phi_p}{\partial z_p}\right] = n_p \frac{\partial \phi_p}{\partial t_p} \quad \ldots\ldots\ldots \quad (2)$$

if the capillary zone is neglected and there is no movement across the free surface (See Appendix I). The same two equations can also be written for the viscous flow in the capillary interspace between two vertical parallel plates of a viscous fluid model. The only difference stems from the fact that the model is isotropic, so that $K_m = \frac{1}{12} g \frac{b^2}{\nu}$. Hence,

$$K_m\left(\frac{\partial^2 \phi_m}{\partial x_m^2} + \frac{\partial^2 \phi_m}{\partial z_m^2}\right) = S_{om} \frac{\partial \phi_m}{\partial t_m} \quad \ldots\ldots\ldots\ldots \quad (3)$$

and

$$K_m\left[\left(\frac{\partial \phi_m}{\partial x_m}\right)^2 + \left(\frac{\partial \phi_m}{\partial z_m}\right)^2 - \left(\frac{\partial \phi_m}{\partial z_m}\right)\right] = n_m \frac{\partial \phi_m}{\partial t_m} \quad \ldots\ldots \quad (4)$$

By substituting

$$\left.\begin{array}{l} K_{xp} = \dfrac{K_{xm}}{K_{xr}}; \quad K_{zp} = \dfrac{K_{zm}}{K_{zr}}; \quad x_p = \dfrac{x_m}{x_r} \\[2ex] z_p = \dfrac{z_m}{z_r}; \quad \phi_p = \dfrac{\phi_m}{\phi_r}; \quad n_p = \dfrac{n_m}{n_r} \\[2ex] t_p = \dfrac{t_m}{t_r}; \quad S_{op} = \dfrac{S_{om}}{S_{or}} \end{array}\right\} \quad \ldots\ldots\ldots \quad (5)$$

into Eq. 2 and comparing the result with Eq. 4, the following relations are obtained according to the laws of similitude:

$$\frac{x_r^2}{K_{xr}\phi_r^2} = \frac{z_r^2}{K_{zr}\phi_r^2} = \frac{z_r}{K_{zr}\phi_r} = \frac{t_r}{n_r \phi_r} = \alpha \quad \ldots\ldots\ldots\ldots \quad (6)$$

Here α can be any number. From the definition of α it follows that $\phi_r z_r$; therefore, the model scales are given by;

$$\frac{x_r^2}{K_{xr}} = \frac{z_r^2}{K_{zr}} \quad \left(\frac{x_r}{z_r}\right)^2 = \frac{K_{xr}}{K_{zr}} = \frac{K_{zp}}{K_{xp}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

and,

$$t_r = \frac{n_r\, z_r}{K_{zr}} \quad \text{or,} \quad t_r = \frac{n_r\, x_r^2}{K_{xr}\, z_r} \qquad (8)$$

LENGTH SCALES

Eq. 7 describes the distortion in length scales in cases of nonisotropic soils. When $K_{xp} = K_{zp}$ the horizontal and vertical scales must necessarily be the same. Very often a model must imitate an aquifer having a length much longer than its depth (for example, several kilometers as compared to a hundred meters). It is impossible to design such a model without distortion of scales. In such cases, it is necessary to assume that the hydraulic conductivity in the horizontal direction is much larger than it is in the vertical direction, and then the distortion of the scales will be determined accordingly. The error which might thus be introduced (in case of a disagreement between the assumed and the existing hydraulic conductivities) is usually small. One has only to remember that the flow is essentially horizontal with a relatively small vertical component due to the elongated shape of the aquifer. However, there are cases, such as in the vicinity of pumping wells or near the outflow into the sea, where this assumption might yield large errors.

TIME SCALES

The choice of the time scale will be governed by the time interval between two successive readings of discharge, levels, etc. Rewriting Eq. 8 in a more detailed form,

$$t_m = \frac{n_m}{n_p} \frac{x_r^2}{z_r} \frac{K_{xp}}{K_{xm}} t_p = \frac{12}{g} \frac{K_{xp}}{n_p} \frac{x_r^2}{z_r} \frac{\nu}{b^2} t_p \qquad (9)$$

This indicates that an appropriate time scale can be obtained for given length scales by manipulating the width of the interspace, b, and the viscosity of the liquid, ν. Consideration of the interrelations between length and time scales will enable the most practical laboratory model to be designed for studying a given problem.

DISCHARGE SCALES

The discharge scale is obtained from Darcy's law. Starting from the prototype flow,

$$Q_{xp} = -\,K_{xp}\, b_p\, z_p \frac{\partial \phi_p}{\partial x_p}$$

and the model flow,

$$Q_{xm} = -\,K_{xm}\, b_m\, z_m \frac{\partial \phi_m}{\partial x_m}$$

it follows that

$$Q_{xr} = \frac{K_{xr}\, b_r\, z_r^2}{x_r} \qquad (10)$$

Similarly,

$$Q_{zr} = K_{zr}\, b_r\, x_r \quad \text{. . . . .} \quad (11)$$

and, it follows that

$$Q_{xr} = Q_{zr} = Q_r$$

VOLUME SCALES

The volume scale is obtained from the continuity equation

$$V_r = Q_r\, t_r = n_r\, x_r\, z_r\, b_r \quad \text{. . . . .} \quad (12)$$

Very often areas of open water such as rivers, lakes, or the sea have to be imitated by a model. In these cases the model consists of three plates (Fig. 1) instead of two. A narrow interspace is kept between two of the plates while the third one, together with cutting the middle plate to the desired shape, serves to increase the interspace up to the sum of the capillary interspace

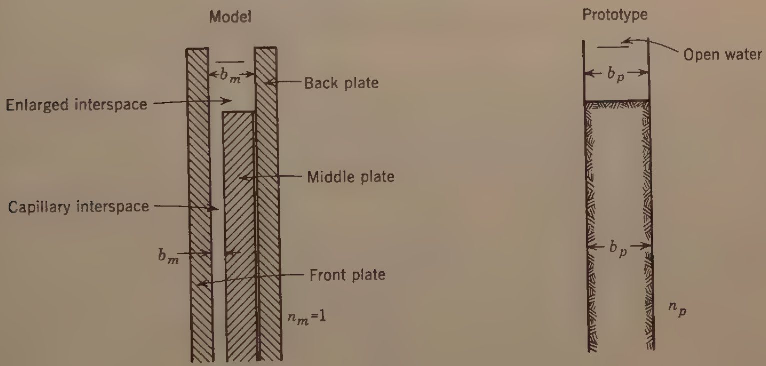

FIG. 1

plus the thickness of the middle plate. When the volume of liquid leaving the reservoir and entering the capillary interspace is of importance, this enlarged interspace is subject to an additional condition:

The same volume scale must be applicable to both the narrow and the enlarged interspaces. With the notation of Fig. 1, the following equations can be written: for the soil

$$V_r = n_r\, x_r\, z_r\, b_r$$

and for the open water

$$\bar{V}_r = \bar{n}_r\, x_r\, z_r\, \bar{b}_r$$

Where the bar indicates values for the open water.

Since

$$V_r = V_r, \qquad \text{and} \qquad \bar{n}_r = 1$$

it follows that

$$n_r b_r = \bar{n}_r \bar{b}_r$$

or,

$$\bar{b}_r = \frac{b_r}{n_p} \quad \text{. . . . .} \quad (13)$$

REPLENISHMENT SCALES

The natural replenishment scale can be derived from the equation which approximately describes the flow in a phreatic aquifer with a horizontal bottom. For the case shown in Fig. 3

$$\frac{\partial}{\partial x}\left(K_x\, h\, \frac{\partial h}{\partial x}\right) + P = n\, \frac{\partial \phi}{\partial t}$$

or, with K_x constant,

$$\frac{1}{2} K_x\, \frac{\partial}{\partial x}\left(\frac{\partial h^2}{\partial x}\right) + P = n\, \frac{\partial \phi}{\partial t}$$

where h is the thickness of the flow and P is the vertical recharge rate. Since $h_r = Z_r$ it follows that

$$P_r = \frac{n_r\ Z_r}{t_r} = K_{zr} = \frac{K_{xr}\ Z_r^2}{x_r^2} \quad \ldots\ldots\ldots\ldots\ (14)$$

NONHOMOGENEOUS AQUIFERS

In case of a nonhomogeneous aquifer, that is, an aquifer which consists of several layers with different permeabilities, an imitation by the model is possible by varying the width of the interspace. It can be done by introducing thin sheets of plastic or any other suitable material with the geometric shape of the layers into the interspace, thereby leaving a narrower space free for the flow of the viscous liquid.

Since the discharge scale must remain the same for the various layers it follows that $K_{zr} b_r$ must remain constant. Hence

$$\frac{K_m}{K_{zp}}\ \frac{b_m}{b_p} = \frac{1}{12}\ g\ \frac{b_m^2}{\nu K_{zp}}\ \frac{b_m}{b_p} = \text{a constant}$$

or

$$\frac{b_m^3}{K_{zp}} = \text{a constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (15)$$

or

$$\frac{b_m^3}{K_{xp}} = \text{a constant}$$

It might be mentioned here that sometimes the condition b^2m/K_{xp} is used instead of Eq. 15. This follows from the assumption that K_{xr} must remain constant throughout the model whereas the right thing is to keep $K_r\ b_r$ constant throughout the layered model.

A condition which causes some trouble when computing scales for a model imitating a layered aquifer is the condition that n_p/b_m should be kept constant throughout the model. This condition follows from Eq. 9 when t_r is kept constant throughout the model. Very often this condition cannot be fulfilled, but the error introduced is generally small. In most practical cases, the most important layers with respect to hydraulic conductivity are the sand or sandstone layers where variations in permeability or in porosity from one layer

to the other are relatively small so that the error introduced by neglecting this last condition is small. However, a larger error will be introduced in the case of more impervious layers.

In the case of an impervious layer, $b_m = 0$; therefore, the entire interspace is filled. When layers of small permeability are present the computed b_m for these layers might come out to be very small (for example, 0.1 mm) and cause difficulties in constructing and operating the model. These cases occur very often when relatively thin semi-pervious layers, in which the flow is mainly in the vertical direction, are present. A practical solution to this problem would be to use strips which completely seal part of interspace and leave a wider interspace in the remaining part as shown in Fig. 2.

The relationship between the partly sealed and the enlarged interspace is determined by the condition that the discharge Q_2 through the whole length in

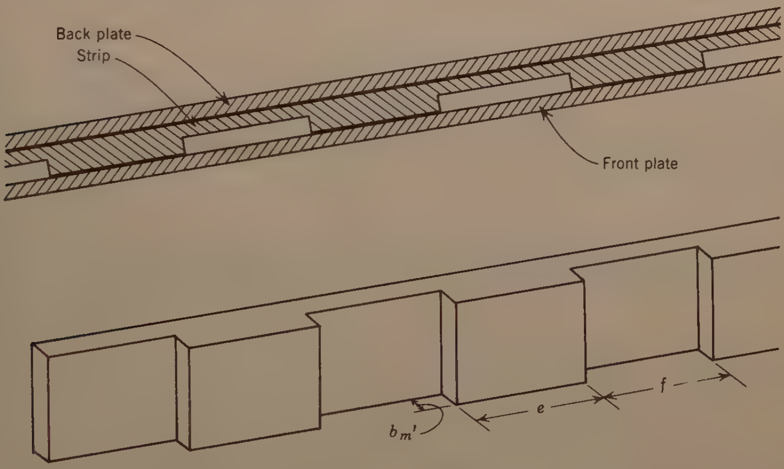

FIG. 2

the case of the narrow interspace b_m must equal the discharge Q_1 through the shortened length with the widened interspace b_m'. Therefore,

$$Q_1 = \frac{1}{12}\, g\, \frac{b_m^2}{\nu}\, (f + e)\, b_m\, \frac{\partial \phi}{\partial x}$$

$$Q_2 = \frac{1}{12}\, g\, \frac{b_m'}{\nu}\, e\, b_m'\, \frac{\partial \phi}{\partial x}$$

from which it follows that

$$b_m^3\, (f + e) = b_m'\, e \qquad (16)$$

Experience shows that in the case of relatively narrow interspaces the free space must be increased by about 15% over the computed values due to increased wall friction. The use of these strips was first introduced by the Hydrological Laboratory of the Government Institute for Water Supply, The Hague.

STORAGE COEFFICIENT CONSIDERATIONS

Up to this point only models which do not imitate storage coefficient have been considered. It was assumed that specific storage coefficient can be

neglected in the case of the phreatic aquifer. In this case the storage coefficient of the aquifer is identical to the effective porosity whereas the specific storage coefficient which results from the elastic properties of soil and water is so small that we can assume $S_{op} = 0$ and $S_{om} = 0$ in Eqs. 1 and 2 and they cannot be used for the determination of the time scale.

The problem of imitating a confined aquifer with specific storage coefficient, S_o, by a model has not yet been completely solved. Nevertheless, there are several possibilities some of which have been successfully tried.

From Eqs. 1 and 3 it follows that the design conditions for a model of a confined aquifer should be:

$$K_{xr} \frac{Z_r}{X_r^2} = \frac{K_{zr}}{Z_r} = S_o \frac{Z_r}{t_r} \qquad (17)$$

or

$$t_r = \frac{S_{or}\ Z_r^2}{K_{zr}} \qquad (18)$$

If it is desired to consider the specific storage coefficient as well as the porosity in the case of an unconfined aquifer, the following condition must also be satisfied:

$$S_{or}\ Z_r = n_r \qquad (19)$$

An aquifer's storage coefficient results from the elasticity of the soil and the water, but since the pressures in the model are relatively small we can-

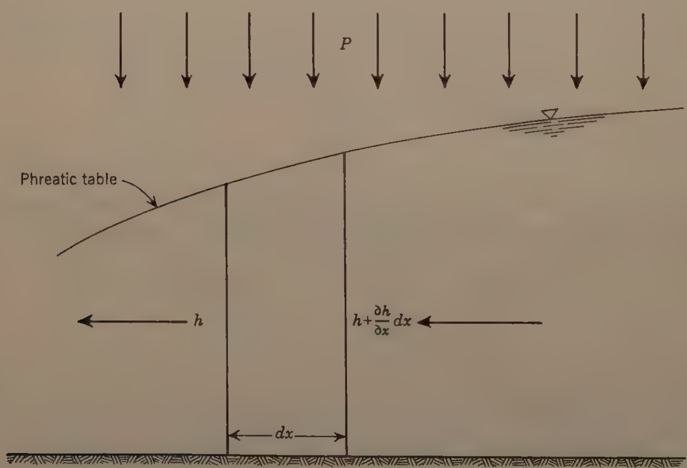

FIG. 3

not use the elasticity of the plates and the viscous fluid for this purpose. The relationship between S and S_o is given by $S = S_o D$, where D is the thickness of the confined aquifer.

One of the several methods used for imitating the storage coefficient of a confined aquifer is to connect to the back plate a vertical pipe the cross section of which will be computed according to the imitated storage coefficient as sketched in Fig. 4. The aquifer length is divided into several parts each with a length of l_m (for example, 10 cm). The storage coefficient of each of these

columns will be represented by a vertical pipe with a cross sectional area a. Hence

$$S_m = \frac{a\ 1.0}{b\ l\ 1.0} = \frac{a}{b\,l} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

and

$$S_r = \frac{S_m}{S_p} = \frac{a}{b\ l\ S_p}$$

When the specific storage coefficient, S_o, varies along the height of the column, it is possible to connect to every opening in the back plate a separate vertical pipe each having a different diameter. In this case the entire area of the plates is divided into rectangles in which the storage coefficient of each is

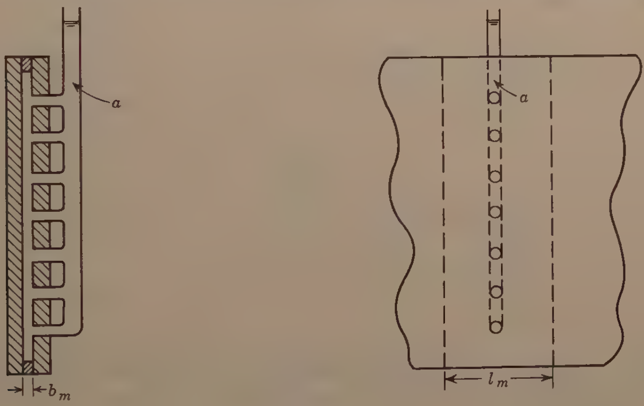

FIG. 4

separately imitated. The same method can be used in order to increase the storage coefficient of a model which imitates a phreatic aquifer so that a more suitable time scale can be obtained. In this case

$$S_m = n_m = \frac{b\ l + a}{b\ l} \quad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

TWO-FLUID FLOWS

An important problem which can be investigated by the model is that of the simultaneous flow of two liquids. The problem of salt water intrusion into coastal aquifers is an example of one of these problems. In a model two viscous liquids (distinguished by different colors) can be used. Certain conditions must be satisfied for this situation.

Assuming the hydraulic conductivity of the model is the same for the areas occupied by the two different liquids, and using the fact that the hydraulic conductivity of the soil is inversely proportional to the viscosity of the water, then

$$K_{mf} = \frac{1}{12}\, g\, \frac{b_m^2}{\nu_{mf}}; \qquad K_{pfx} = C\,\frac{1}{\nu_{pf}}$$

$$K_{ms} = \frac{1}{12}\, g\, \frac{b_m^2}{\nu_{ms}}; \qquad K_{psx} = C\,\frac{1}{\nu_{ps}}$$

and
$$K_{rf} = K_{rs}$$

Hence,
$$\left(\frac{\nu_m}{\nu_p}\right)_f = \left(\frac{\nu_m}{\nu_p}\right)_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

where subscript f indicates the fresh water and subscript s indicates the salt water.

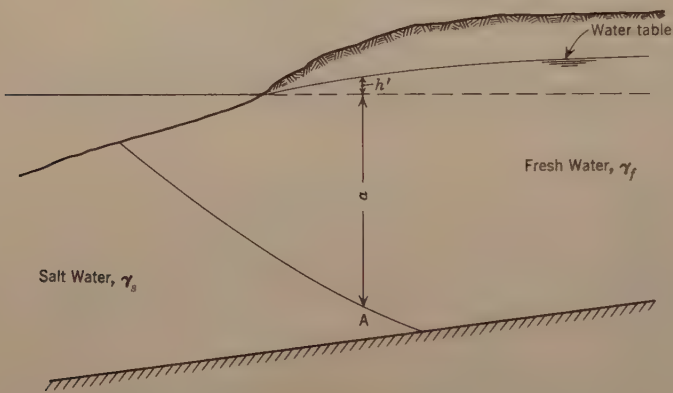

FIG. 5

The pressure at any point along the interface is the same on both sides of the interface. In Fig. 5, showing a coastal phreatic aquifer, at point A the equation for the prototype is

$$a_p \, \gamma_{sp} = (a_p + h'_p) \, \gamma_{fp}$$

or
$$a_p \, \delta_p = h'_p$$

where
$$\delta = \frac{\gamma_s - \gamma_f}{\gamma_f}$$

Similarly, for the model

$$a_m \, \delta_m = h'_m$$

and hence
$$a_r \, \delta_r = h'_r$$

Because
$$a_r = h'_r = z_r \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

it follows that the choice of the two liquids will be governed by the condition that:

$$\delta_r = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

so that

$$\left(\frac{\gamma_f - \gamma_s}{\gamma_f}\right)_m = \left(\frac{\gamma_f - \gamma_s}{\gamma_f}\right)_p \quad \ldots\ldots\ldots\ldots \quad (26)$$

CAPILLARY CONSIDERATIONS

The problem of the capillary rise of the viscous liquid which is flowing in the interspace must also be considered. The water table of this liquid does not represent a phreatic surface (on which the pressure is the atmospheric pressure), but the surface which results from the capillary rise. The height of the capillary rise depends upon the width of the interspace, the type of liquid, and the material of the plates. The phreatic table in the model is obtained by deducting the capillary rise between the plates from the liquid surface elevation.

In ground water problems where the thickness of the saturated layers is much larger than the capillary rise, this phenomenon is of secondary importance only. Yet, by choosing a suitable interspace, liquid, and vertical scale, the capillary rise in the model can be made to correspond to the capillary rise in the soil.

There are other problems, such as seepage from reservoirs and channels in the first stages of the non-steady flow, where the capillary effect (in all directions) is of primary importance. Corrections can be made by a proper change in the driving head in the reservoir or the channel.

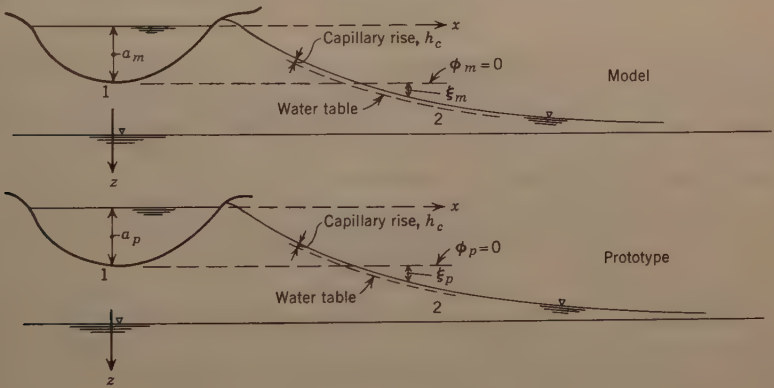

FIG. 6

As an example, consider the flow from an artificial replenishment reservoir (two-dimensional case) shown in Fig. 6. The potential at point 1 is:

$$\phi_1 = a + a = 0$$

The potential at point 2 on the wetting front is: $\phi_2 = (a + \zeta) - h_c$

where h_c denotes the capillary rise and ζ the ordinates of points on the water surface. The same expressions can be written both for the model and the prototype:

$$\phi_{1m} = 0$$
$$\phi_{2m} = -(a + h_c + \zeta)_m$$
$$\Delta\phi_m = \phi_{1m} - \phi_{2m} = (a + h_c + \zeta)_m = (a + h_c)_m + \zeta_m$$

$$\phi_{1p} = 0$$
$$\phi_{2p} = -(a + h_c + \zeta)_p$$
$$\Delta\phi_p = \phi_{1p} - \phi_{2p} = (a + h_c + \zeta)_p = (a + h_c)_p + \zeta_p$$

It follows that

$$\frac{\Delta\phi_m}{\Delta\phi_p} = \frac{\zeta_m}{\zeta_p} = \frac{(a + h_c)_m}{(a + h_c)_p} = Z_r \quad \ldots\ldots\ldots\ldots \quad (27)$$

This means that the water surface in the model reservoir must be raised so that

$$(a + h_c)_m = (a + h_c)_p$$

When raising this level, special attention must be given to the fact that the geometric boundaries through which the seepage takes place are not altered.

ACKNOWLEDGMENT

The author wishes to acknowledge the experience in ground water model work gained with Ir. G. Santing in the Government Institute for Water Supply, The Hague, and to thank Professor David K. Todd for reviewing the draft of this paper and for his helpful suggestions.

APPENDIX I

Let $F(x,y,z,t) = 0$ be the function which describes the phreatic surface. If $P(x,y,z)$ is a particle on this surface at time t and $P(x + \Delta x, y + \Delta y, z + \Delta z)$ is the position of the same particle at time $t + \Delta t$ when the phreatic surface is moving, then $F(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) = 0$. The change can be expressed by the total derivative in the form:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial t}$$

The potential $\phi = z + \frac{p}{\gamma}$ for the phreatic surface $(p = 0)$ takes the form $\phi = z$, or $F = \phi - z = 0$.

Hence,

$$\frac{DF}{Dt} = 0 = \frac{\partial \phi}{\partial t} - \frac{\partial z}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \phi}{\partial z}\frac{\partial z}{\partial t} \quad \ldots\ldots \quad (1)$$

Now $\frac{\partial x}{\partial t}$, $\frac{\partial y}{\partial t}$, and $\frac{\partial x}{\partial t}$ describe the components of the velocity u, v, w in the x, y, and z directions, respectively.

According to Darcy's laws

$$\underline{q} = - K \operatorname{grad} \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

hence

$$\left.\begin{aligned} q_x &= - K_x \frac{\partial \phi}{\partial x} = n_u \\ q_y &= - K_y \frac{\partial \phi}{\partial y} = n_v \\ q_z &= - K_z \frac{\partial \phi}{\partial z} = n_w \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where n is the effective porosity and all subscripts indicate directions.

By inserting (3) into (1), the resulting final equation is

$$K_x\left(\frac{\partial\phi}{\partial x}\right)^2 + K_y\left(\frac{\partial\phi}{\partial y}\right)^2 + K_z\left[\left(\frac{\partial\phi}{\partial z}\right)^2 - \left(\frac{\partial\phi}{\partial z}\right)\right] = n\,\frac{\partial\phi}{\partial t}$$

APPENDIX II—SYMBOLS

a = pipe area;
b_m = width of interspace;
b_p = width of investigated area of prototype;
g = acceletation of gravity;
h = capillary rise;
K^c = hydraulic conductivity;
K = hydraulic conductivity in the x-direction;
K_z = hydraulic conductivity in the z-direction;
m = subscript, indicates model values;
P = vertical recharge rate;
p = pressure;
p = subscript, indicates prototype values;
Q = discharge;
r = subscript, indicates the model-prototype ratio;
S_o = specific storage coefficient defined as the volume of water released from storage in a unit volume of aquifer by a unit decline of head; dimension L;
S = storage coefficient, defined as the volume of water released from a column of aquifer (the height of the whole aquifer) having a unit cross-sectional area by a unit decline of head. It is dimensionless;
t = time;
V = volume;
x, y, z = coordinate axes;
α = constant;
γ_s, γ_F = specific weight of salt and fresh water, respectively;
δ = ratio of the difference between the specific weights of salt and fresh water to the specific weight of fresh water;
ν = kinematic viscosity; and
ϕ = ground water potential at every point.

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BOUNDARY LAYER STIMULATION IN RECTANGULAR CONDUITS

By R. G. Cox,[1] M. ASCE and F. L. Bauer[2]

SYNOPSIS

The effects of artificial stimulation of the turbulent boundary layer were studied in a 0.283-ft by 0.5-ft plastic conduit. The length of conduit required for fully developed pipe flow was reduced from 18.5-ft to 14-ft by 1/8-in. cubes cemented to the conduit entrance curves.

INTRODUCTION

This paper describes an investigation of the effects of artificial stimulation of the turbulent boundary layer in rectangular conduits and indicates the ultimate use of boundary layer stimulation in models of hydraulic structures.

A review of the problem resulting in the study is of interest. For the past seven years the Waterways Experiment Station has cooperated with the United States Army Engineer Districts in field tests on large hydraulic structures, a number of which have been made on sluices and conduits of flood control dams. The elements of a typical conduit, consisting of the curved entrance, the control gate section, and the conduit proper, are shown on Fig. 1 for a concrete gravity dam. Conduits may also contain transitions downstream from the gate section, vertical and horizontal bends, and exit portal constrictions. As flow passes through each section, it loses energy. For convenience, the energy losses occurring upstream from the uniform conduit section are grouped together and termed the intake loss. This loss is considered to be the total

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

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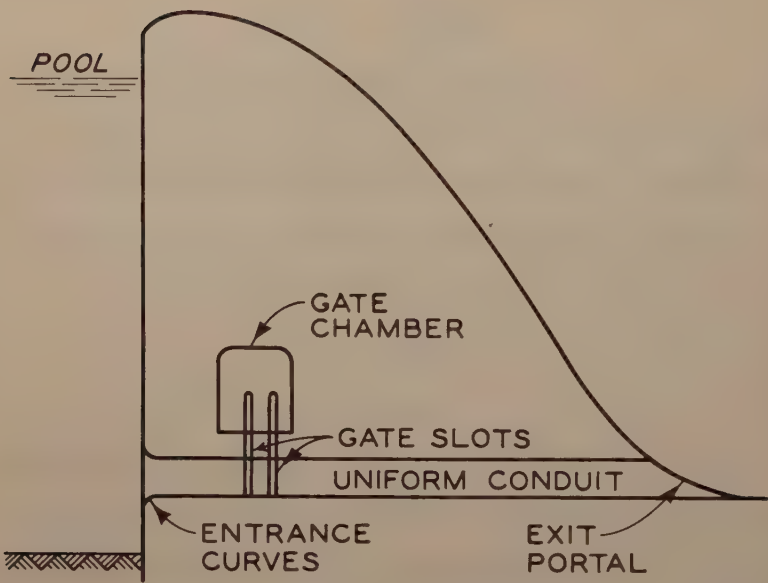

FIG. 1.—ELEMENT OF A TYPICAL CONDUIT

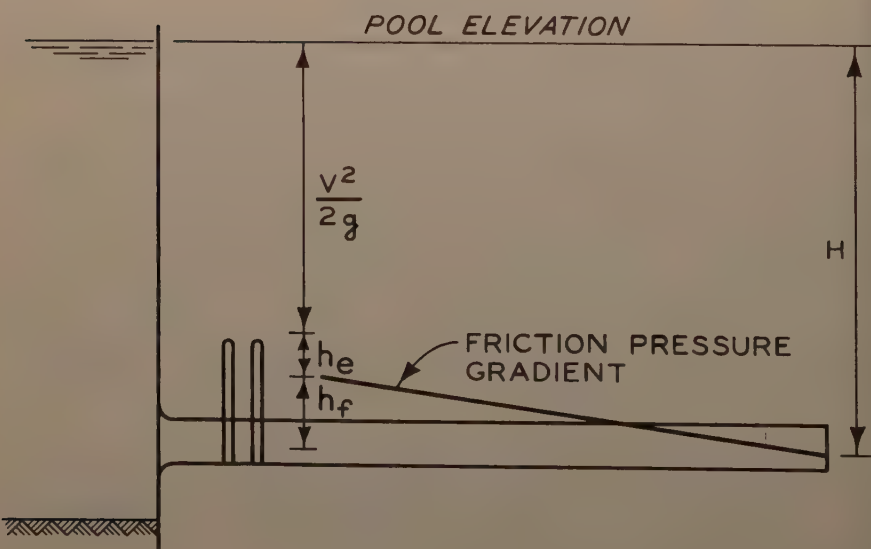

FIG. 2.—ENERGY LOSSES IN A TYPICAL CONDUIT

available energy head H minus the conduit velocity head and the friction loss h_f and miscellaneous losses in the uniform conduit section (Fig. 2). The intake loss, as defined, includes the losses attributable to the entrance curves, to the gate section and slots, and to the tunnel transition. It may be expressed as a function of a dimensionless coefficient by

$$K_e = \frac{h_e}{V^2/2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which the energy losses (intake) $h_e = H - h_f - \frac{V^2}{2g}$ and $V^2/2g$ is the velocity head (conduit proper). In every case where comparison between model and prototype results has been possible, the prototype loss coefficient is considerably greater than that observed in the model. For example, the observed Pine Flat Dam sluice prototype coefficient is 0.16. A comparable laboratory study indicates a coefficient of 0.07. Similar results were noted for Denison Dam flood control conduits where coefficients of 0.12 in the model and 0.19 in the prototype were observed.

Models of hydraulic structures are usually designed on the basis of geometric similitude and Froude's law. The latter requires that the inertial and gravitational forces have the same relation in the model as in the prototype. Consequently, the model will have a smaller Reynolds number than the prototype and relatively greater friction losses. Therefore, the model is made as smooth as possible to obtain a closer friction relation to the prototype. It is believed that the turbulent boundary layer may develop more rapidly at the very high Reynolds numbers that occur in the prototype than at the low Reynolds numbers that occur in the model. Further, the entrance, gate slots and transition of the prototype could establish a higher turbulent energy level than could be sustained by wall roughness alone. In the case of the model, the boundary layer may develop more slowly and may not reach a turbulent energy level comparable to that of the prototype. If such is the case, artificial stimulation of turbulence in models may be necessary to reproduce the intake loss coefficient of the prototype.

In an attempt to understand the reasons for the apparent differences between model and prototype intake losses, the Waterways Experiment Station has undertaken three approaches to the problem. First, theoretical studies of boundary layer development and wall shear stress on curved surfaces were initiated, and are still in progress. Second, in cooperation with the Omaha District, a horizontal strut was installed in one of the 22-ft-diameter flood control tunnels of Ft. Randall Dam. This strut contained numerous instruments for measuring velocity distribution and turbulence fluctuations. Tests were made with flow up to 50 fps. In addition, a specially designed probe was used to investigate velocity distribution within one foot of the tunnel wall. And third, a laboratory study of the effects of artificial stimulation of the turbulent boundary layer on flow in a rectangular conduit was made. The latter study forms the basis for this paper.

PREVIOUS STUDIES OF ARTIFICIAL STIMULATION

For a number of years, engineers engaged in testing the drag on ship models at the David Taylor Model Basin, U. S. Navy, have artificially stimulated the boundary layer to compensate for the relative difference in boundary layer

growth in the model and prototype. Breslin and Macovsky[3] report that without stimulation, this difference results in appreciable error in the predicted power requirements of the prototype at design speeds. This error has been attributed to the fact that the slower development of the turbulent boundary layer at the low model Reynolds number results in smaller shearing stresses than does the more rapid development at the high prototype Reynolds number. Prior to 1950 there was but little published literature describing attempts to investigate the effects of artificial stimulation on the boundary layer itself. The work of Breslin and Macovsky was a pioneer study in this field.

THEORY OF TURBULENT BOUNDARY LAYER

The theory of the turbulent boundary layer has been presented in numerous publications. Much work has been done on boundary layer development on flat plates and in circular conduits, and a limited amount on open channels of large

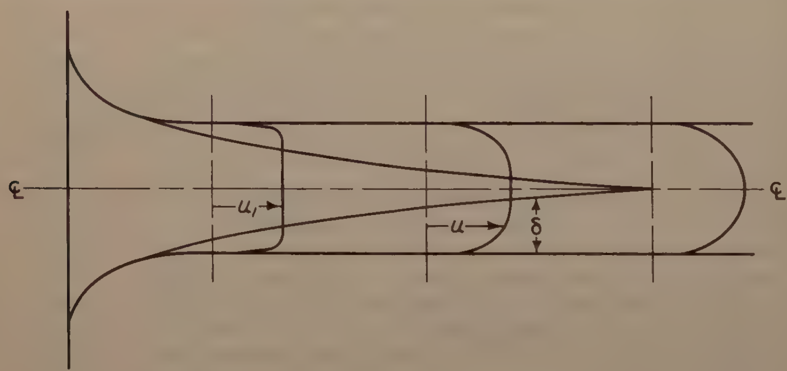

FIG. 3.—BOUNDARY LAYER DEVELOPMENT

width-depth ratios. An extensive search of published literature revealed no record of any studies conducted on rectangular conduits of varying ratios of height to width.

A brief summary of the theory of boundary layer development in a conduit is pertinent to the investigation to be described. It is generally known that in pipes the flow is laminar with Reynolds numbers less than 2,000; that a transition range occurs between 2,000 and 3,000; and that above a Reynolds number of 3,000 the flow is turbulent.

Boundary Layer Parameters.—Turbulent flow in a conduit develops by a progressive thickening of the boundary layer in the region close to the wall until this thickness is equal to the radius of the conduit at which point complete turbulence is established. In the case of the rectangular conduit this thickness

3 "Effects of Turbulence Stimulators on the Boundary Layer and Resistance of a Ship Model as Detected by Hot Wires," by J. P. Breslin and M. S. Macovsky, without stimulation, this difference results in appreciable error in the Navy Department, David Taylor Model Basin Report 724, August 1950.

would be one-half of the minimum conduit dimension. This phenomenon, for two-dimensional flow, is illustrated by Fig. 3, in which u_1 is the velocity in the undisturbed central core; u represents the longitudinal component of the velocity within the turbulent boundary layer; and δ denotes the thickness of the turbulent boundary layer.

Rouse[4] defines the nominal thickness of the turbulent boundary layer δ as the thickness of the zone in which the major portion of the velocity variation takes place. Velocity distribution in the nominal boundary layer is illustrated by Fig. 4.

Two important parameters used in the study of the turbulent boundary layer are the displacement thickness[4] δ and the momentum thickness θ. The displacement thickness is usually defined as the distance which the streamlines

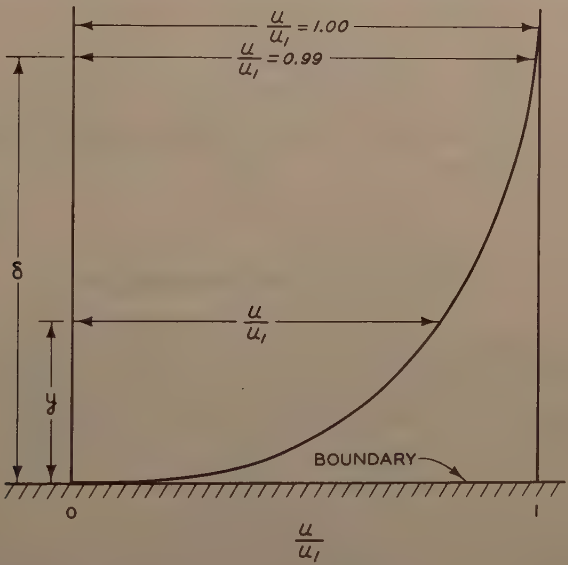

FIG. 4.—VELOCITY DISTRIBUTION IN THE BOUNDARY LAYER

have been displaced from the boundary as a result of the turbulent boundary layer. The displacement thickness[4] can be expressed as:

$$\delta = \int_0^{\delta} \left(1 - \frac{u}{u_1}\right) dy \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

A clearer concept of this parameter has been presented by Lighthill[5] who states

> ". . . between the surface and any streamline just outside the boundary layer, there must be a constant volume of flow per unit span. This will be so if the flow reduction inside the layer is compensated for by an outward displacement of such a streamline through a distance δ_1 (which

4 "Engineering Hydraulics," by H. Rouse, John Wiley and Sons, Inc., N. Y., 1950.

5 "On Displacement Thickness," by M. J. Lighthill, Cambridge University Press, Journal of Fluid Mechanics, Vol. 4, Part 4, August 1948.

produces a flow increase $U\,\delta_1$, since the velocity is U in the region of streamline displacement)."

In the foregoing, δ_1 is the displacement thickness and U is the streamline velocity.

The momentum thickness θ is generally defined as a measure of the defect of the momentum flux or decrease of energy within the boundary layer. The momentum thickness can be expressed as:

$$\theta = \int_0^{\delta} \frac{u}{u_1}\left(1 - \frac{u}{u_1}\right) dy \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

Fig. 5 illustrates the physical and mathematical significance of these two parameters.

Other useful parameters in the study of the boundary layer are the local momentum Reynolds number:

$$R_\theta = \frac{\theta u_1}{\gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

in which γ is the kinematic viscosity of the fluid; and the local resistance coefficient as defined by Ross:[6]

$$C_f = (4.4 + 3.81 \log R_\theta)^{-2} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

DESCRIPTION OF TEST APPARATUS

Tests were conducted on a rectangular plastic conduit 0.283 ft wide, 0.5 ft high and 30 ft long. The axis of the conduit was mounted perpendicular to the vertical face of a pressure tank. The transition from the pressure tank to the conduit was made by four elliptical surfaces formed to the equation:

$$\frac{X^2}{(0.75D)^2} + \frac{Y^2}{(0.25D)^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which D is equal to the dimension of the conduit in the direction concerned.

Flow was controlled by regulating valves and measured by venturi meters located between the pressure tank and the water supply system. The pressure tank head was measured by a mercurial U-tube manometer and referenced to the center line of the conduit. The conduit pressure gradient was determined by side center-line piezometers in the conduit proper connected to a manometer bank. Fig. 6 is a schematic plan of the general test installation shown in Fig. 7.

Horizontal traverses at selected stations were made by means of a stagnation tube. This tube consisted of a traversing rod 0.25-in. in diameter to which was fixed a 0.5 in.-long hypodermic needle, 0.028-in. O.D. and 0.016 in. I.D. The traversing rod and hypodermic needle were inserted in the conduit at the selected station prior to beginning of operation. Leakage around the traversing rod was controlled by means of packing glands. Fig. 8 shows the traversing rod and hypodermic needle inserted in the conduit.

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6 "Turbulence Flow in the Entrance Region of a Pipe, by D. Ross, Paper No. 54-A-89, ASME Preprint, 1955.

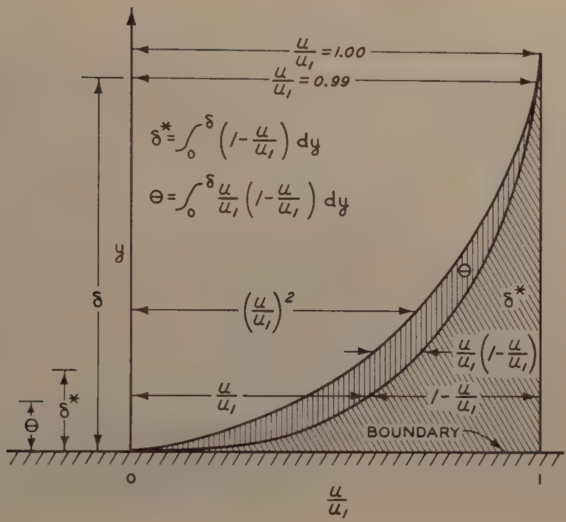

FIG. 5.—DISPLACEMENT THICKNESS δ AND MOMENTUM THICKNESS θ

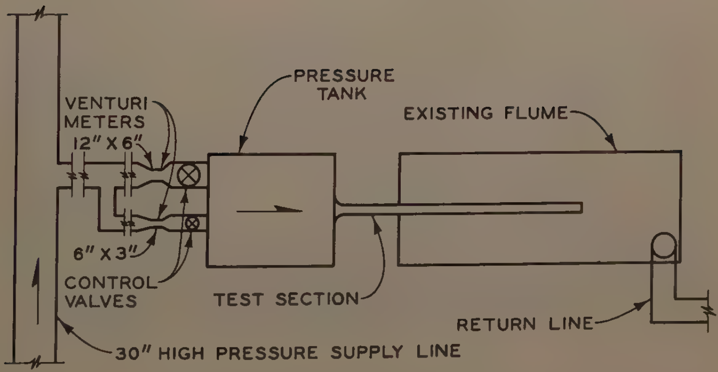

FIG. 6.—PLAN OF TEST INSTALLATION

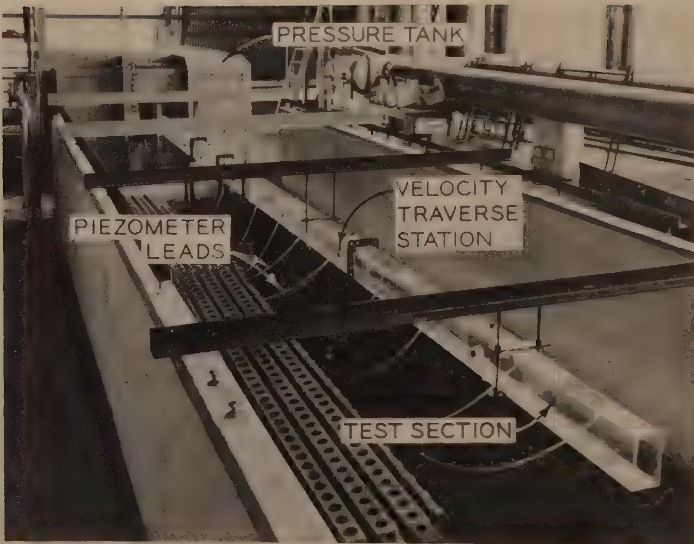

FIG. 7

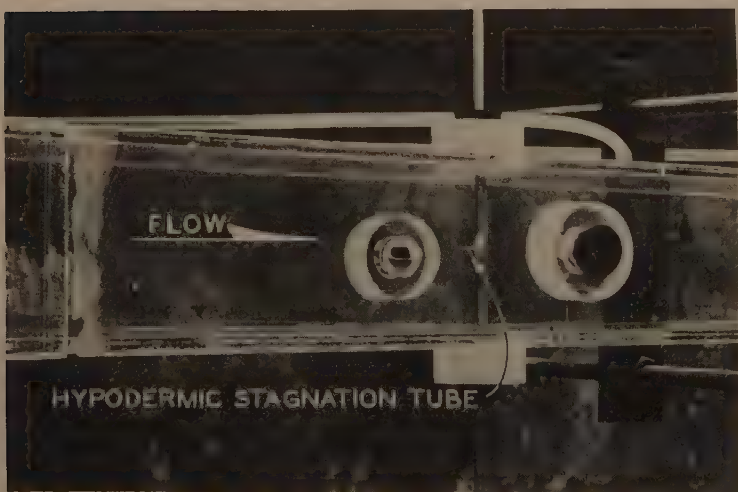

FIG. 8.—STAGNATION TUBE

The boundary layer was stimulated by blocks cemented to the curved entrance as shown by Fig. 9. The blocks were 0.02-ft square with 0.02-ft clearance between each block. Separate tests were made on 1/16-in. and 1/8-in. high blocks. In each test the stimulators were fastened to all four surfaces of the entrance curves.

TEST PROCEDURE

The procedure followed was identical for each of the three series of tests (no stimulation; 1/16-in. stimulation, and with 1/8-in. stimulation). Tests were

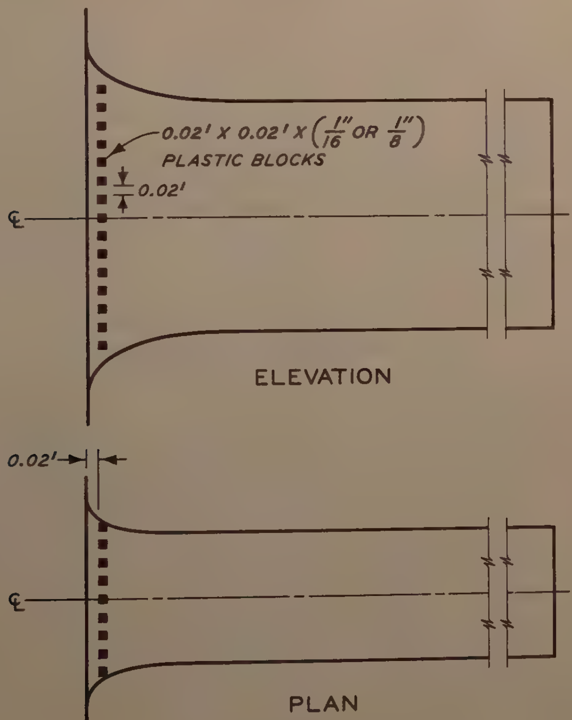

FIG. 9.—ARTIFICIAL STIMULATION

made with heads of 10 ft and 13 ft of water in the pressure tank for each degree of stimulation. The resulting discharge through the conduit was determined. The conduit side center-line piezometric pressures were read on the manometer bank to the nearest 0.005 ft. Velocity distribution data in the boundary layer were obtained by traversing the stagnation tube across the conduit. The first two readings were taken at 0.005-ft intervals from the conduit wall. Successive readings were made at 0.01-ft intervals to the centerline of the conduit. The tube was always oriented to permit determination of the x-velocity component. Fig. 10 shows the piezometer locations. The velocity traverse stations are shown on Fig. 11.

DISCUSSION OF DATA

In the interest of brevity, the discussion of data is limited to tests on flows having a Reynolds number of 6.02×10^5. This Reynolds number was computed using the diameter of a pipe equal to four times the hydraulic radius of the test

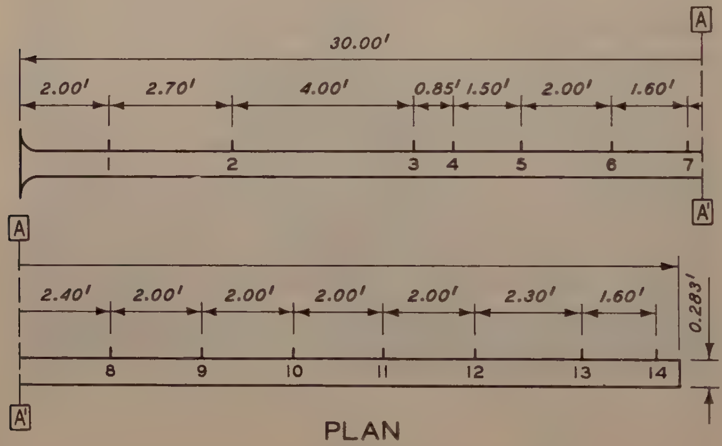

FIG. 10.—PIEZOMETER LOCATIONS

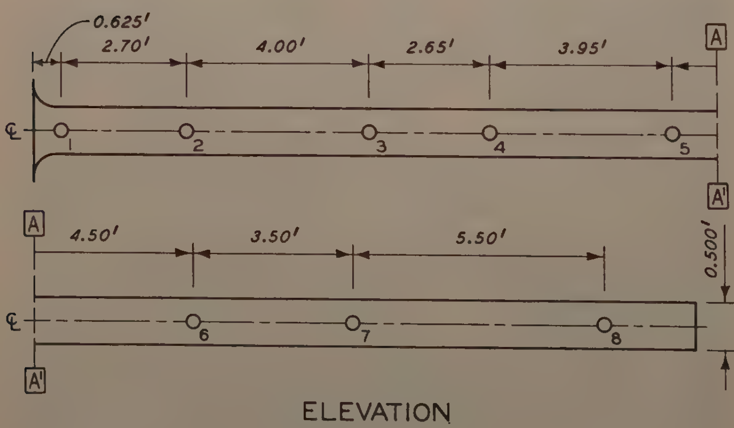

FIG. 11.—VELOCITY TRAVERSE STATIONS

conduit. The results of tests on flows with other Reynolds numbers were similar to the results presented in this paper.

Velocity Data.—The local static pressure at each traversing station was computed from the pressure data measured at the side center-line piezometers. The stagnation tube and the local static pressure data were used to compute the velocity distribution across the conduit. A semi-log plot of the velocity data

with respect to the conduit wall was made. Two straight lines could be drawn through each set of plotted data: a sloping line starting at the conduit wall and a line perpendicular to the conduit wall. The sloping line defines the logarithmic law of the velocity distribution in the turbulent boundary layer and the perpendicular line the velocity in the undisturbed core. The boundary layer thickness was determined graphically by the intersection of these two lines.

Velocity traverses for unstimulated flow are shown on Fig. 12. The progressive thickening of the boundary layer from station 1 to station 4 is shown

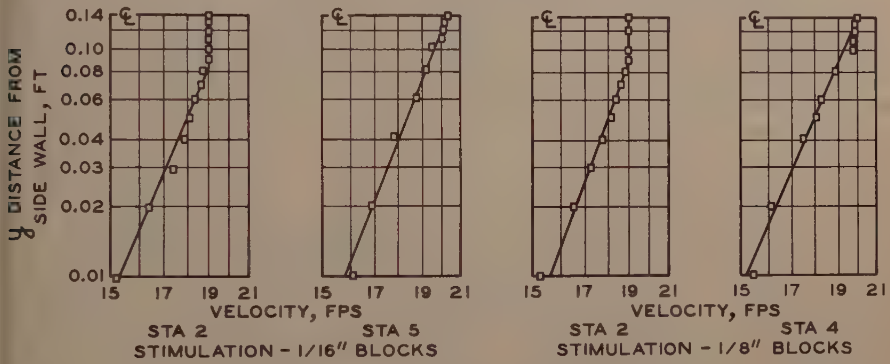

FIG. 12.—BOUNDARY LAYER UNSTIMULATED

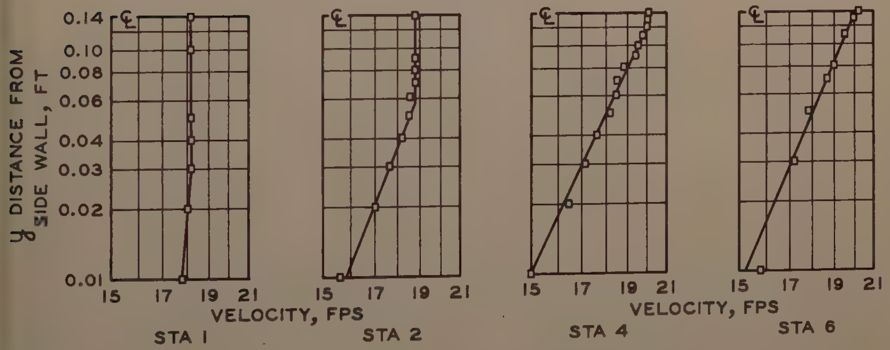

FIG. 13.—BOUNDARY LAYER STIMULATED

by the position of the intersection of the velocity profile lines. The absence of the constant velocity line at station 6 indicates that the boundary layer is fully developed. This station is located about 50 equivalent diameters downstream from the entrance.

Velocity traverses for stimulated flow are shown on Fig. 13. With 1/16-in. block stimulation, the sloping line at station 5 extends to the center line of the conduit. An almost similar condition exists at station 4 with 1/8-in. block stimulation.

The boundary layer thicknesses at each station was determined from the plots of the velocity profiles. These data, plotted on Fig. 14, show the progressive thickening of the turbulent boundary layers with and without stimulation. The effects of successive increases in the degree of stimulation are shown by the inward movement of the boundary layer and the decrease in conduit length required for fully developed turbulence.

Pressure Data.—The observed conduit pressure profile without boundary layer stimulation is shown on Fig. 15 (pressures referenced to side center line). This profile indicates that the friction pressure gradient extends to within 11 ft of the conduit entrance. The steeper portion of the pressure gradient upstream from this point occurs in the region of the conduit where the boundary layer is rapidly developing. The effect of stimulation on the pressure gradient is shown on Fig. 16 (pressures referenced to side center line). With each increase in the degree of stimulation, a corresponding decrease occurs in the elevation of

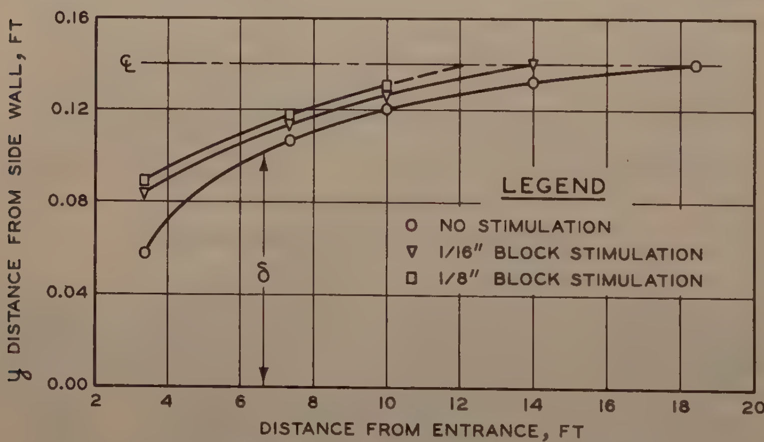

FIG. 14.—STIMULATION AND BOUNDARY THICKNESS

the pressure gradient in the region of boundary layer development together with a slight upstream extension of the friction pressure gradient line.

Friction.—The observed pressure gradient in the last 19 ft of the conduit was used to compute a friction factor by the Darcy-Weisbach formula. For flow with a Reynolds number of 6.02×10^5 an "f" value of 0.011 was obtained. This value plots below the smooth pipe curve on the Moody-type friction diagram.

The equation for the wall-shear-stress coefficient, developed by Ross

$$C_f = (4.4 + 3.81 \log R_\theta)^{-2} \quad \ldots \ldots \ldots \ldots \ldots \quad (7)$$

for circular pipes was used to show qualitative effects of stimulation on the wall shear stress. Application of this equation to the data from the unstimulated boundary layer resulted in a decrease in the wall-shear-stress coefficient from 0.0027 at station 2 to 0.0022 at station 6 where the boundary layer was fully developed. With maximum boundary layer stimulation the coefficients were found to decrease from 0.0026 at station 2 to 0.0023 at station 4, again

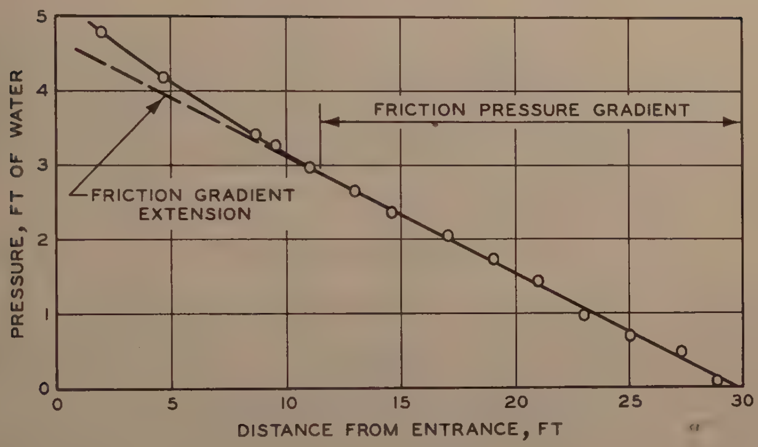

FIG. 15.—PRESSURE PROFILE - NO STIMULATION

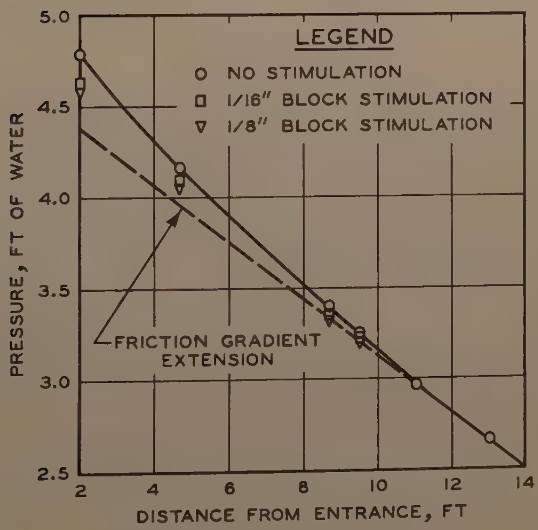

FIG. 16.—EFFECTS OF STIMULATION ON PRESSURE PROFILES

where the boundary layer was assumed fully developed. While the Ross equation was developed from data for circular pipes, its application to a rectangular conduit is considered valid.

Intake Losses.—The empirical intake loss coefficient defined earlier in this paper is dependent upon extension of the friction gradient upstream to the intake structure. A slight decrease in the slope and elevation of the friction pressure gradient was noted with maximum stimulation. It is believed that this resulted from an increase in the intake loss and a decrease in discharge too small to measure with available equipment. It is hoped that other tests can be made with greater stimulation and more refined equipment.

CONCLUSIONS

The following conclusions can be drawn from the investigation:

a. Unstimulated fully developed pipe flow will occur at about 50 equivalent diameters from the entrance of rectangular conduits with a height-width ratio of 1.77 when operated in the range of Reynolds number of 10^6.

b. Stimulation of the boundary layer will effectively reduce the conduit length required for fully developed turbulent flow to about 40 equivalent diameters.

c. The friction pressure gradient indicates fully developed turbulence at about 30 diameters from the entrance.

d. Computation of intake losses based on pressure gradients within 30 diameters of the entrance will result in erroneous indications of the intake losses.

ACKNOWLEDGMENTS

The investigation described was sponsored by the Office, Chief of Engineers under Civil Works Investigation 802, "Conduit Intake Model Tests," during the period January to August 1955. The investigation was planned as a Junior Engineer Officer training program for 1st Lt. (now Captain) Frank L. Bauer. The study was made at the Waterways Experiment Station under the supervision of Mr. F. B. Campbell, Chief, Hydraulic Analysis Branch. Messrs. F. R. Brown and T. E. Murphy of the Hydrodynamics Branch were technical advisors during the course of the study.

The official report[7] prepared by the junior author has been used extensively by the senior author for this presentation. Much credit is due Captain Bauer for his work in the field of artificial stimulation of the turbulent boundary layer in models of hydraulic structures.

7 "The Effects of Artificial Stimulation of the Turbulent Boundary Layer in Rectangular Conduits," Waterways Experiment Station, Miscellaneous Paper 2-160, March 1956.

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

SCOUR AT BRIDGE CROSSINGS

By Emmett M. Laursen,[1] M. ASCE

SYNOPSIS

Relationships are proposed for the prediction of scour at piers and abutments for the case in which sediment is supplied to the scour hole. The relationships were obtained from a combination of an approximate analysis and laboratory experiments, and depend on knowledge of the flow conditions at the bridge site.

GENERAL ASPECTS OF THE PROBLEM

In order to design the foundations of a bridge over an alluvial stream, it is necessary to know the elevation of the stream bed in the vicinity of the piers and abutments. The elevation in question, however, is not the elevation at the time a survey happened to be made, but the lowest elevation which will occur during the anticipated life of the bridge. Unfortunately, engineers are equipped with slide rules rather than crystal balls; therefore this lowest elevation must be coupled with a probability of occurence. On this basis an economic analysis is possible with the cost of construction for one stream bed elevation lower than another considered as an insurance premium for the decreased chance of loss (including interruption of traffic).

Causes of Scour at Bridge Piers and Abutments.—A lowering of the stream bed in the vicinity of the piers and abutments can occur from a variety of causes. A useful distinction can be made by separating the various causes into two general categories; (1) those characteristic of the stream itself, and

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

[1] Assoc. Prof., Dept. of Civ. Engrg., Michigan State Univ., East Lansing, Mich.

(2) those due to the modification of the flow by the bridge crossing. For the past ten years scour due to the modification of the flow has been studied at the Iowa Institute of Hydraulic Research under the sponsorship of the Iowa State Highway Commission and the Bureau of Public Roads. As a result of that investigation means for predicting the scour at bridge piers have been evolved. The relationships which were obtained will be presented in skeleton form in a later section; for a full discussion of the investigation and the design relationships, reference should be made to the final reports which have been published in bulletin form by the Iowa Highway Research Board.[2,3]

Even without the complicating feature of a bridge crossing the bed and banks of an alluvial stream cannot be considered as fixed. Superposed on the very slow changes measured in geological time, there are changes which may occur suddenly in a single flood, periodically during a series of water years, or slowly during the life of a bridge. Past observations at the chosen site, or at similar sites, must be relied upon to estimate the scour which may occur in the future. Each river and each reach must be studied to understand its individual, almost personalized, characteristics.

The reach which includes the chosen bridge site may be degrading: naturally, as the erosional agent of the geological cycle, as a consequence of a dam some distance upstream, or as the result of stream straightening downstream. Old surveys and other records should yield evidence of natural degradation which can be used to predict the future. Degradation caused by the works of man can be analyzed, at least approximately, on the basis of sediment-transport relationships, that is, if the plans of man can be anticipated.

Meandering streams will tend to have the greatest depth at the outside of the bend and shallows between the bends. During high water, the bends will scour and the crossings fill; during low water, the roles will reverse. As the loops of the meanders grow, there may be natural cutoffs with resultant degradation of the reach above and aggradation of the reach below. Usually the meander grows, scouring the outside of the bend, filling the inside; it is not unusual during a flood, however, for a chute to develop across the inside of a bend and even replace the old channel.

Braided streams are characteristically wide and shallow, with deeper channels which may shift erratically. Even if it is not believed that "if anything can go wrong, it will" is a natural law, it must be admitted that there is a fair chance that during the life of a bridge a deep channel may develop at a pier at the time of a flood.

Any contraction, whether of the main channel or of the overbank flow, will result in scour during high stages. As shown by L. G. Straub,[4] the principle of continuity applied to both the discharge and the sediment load permit this case to be solved analytically since the flow conditions in both the contracted and uncontracted reaches can be considered uniform. It is likely that this case of scour has given rise to the old riverman's belief that during a flood a plains river will scour its bed as much as the water surface rises. That such

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2 "Scour Around Bridge Piers and Abutments," by E. M. Laursen, and A. Toch, Iowa Highway Research Board Bulletin No. 4, May 1956.

3 "Scour at Bridge Crossings," by E. M. Laursen, Iowa Highway Research Board Bulletin No. 8, August 1958.

4 "Approaches to the Study of Mechanics of Bed Movement," by L. G. Straub, Proceedings (First) Hydraulics Conference, State University of Iowa, Iowa City, Iowa, 1940.

scour cannot be general throughout the stream has been well demonstrated by E. W. Lane and W. M. Borland.[5]

The Long Contraction.—By describing the conditions in the uniform reaches above and in a long contraction by a discharge equation and a sediment-transport equation, it is possible to solve for the ratio of the depths of flow in the two reaches—and, therefore, the depth of scour. The solution will depend, in detail, upon the equations selected to describe the flow and transport. Fig. 1 is a definition sketch of the long contraction in which there is both channel and overbank contraction. The Manning formula can be used to describe the flow conditions

$$Q_c = \frac{1.49}{n_1} B_1 \, y_1^{5/3} \, S_1^{1/2} \quad \ldots\ldots\ldots\ldots \quad (1)$$

and

$$Q_t = \frac{1.49}{n_2} B_2 \, y_2^{5/3} \, S_2^{1/2} \quad \ldots\ldots\ldots\ldots \quad (2)$$

An approximate form of the total-sediment-load relationship recently proposed[6] can be used to describe the sediment concentration

$$\bar{c} = \left(\frac{D}{y}\right)^{7/6} \left(\frac{\tau_0'}{\tau_c} - 1\right) b \left(\frac{\sqrt{\tau_0/\rho}}{w}\right)^a \quad \ldots\ldots\ldots \quad (3)$$

in which

$$\frac{\tau_0'}{\tau_c} = \frac{V^2}{120 y^{1/3} D^{2/3}} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

and

$$\sqrt{\tau_0/\rho} = \sqrt{g y S} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

That portion of the shear associated with the sediment particle of size D is termed τ_0' and τ_c is the critical shear for sediment movement. The coefficient b and the exponent a depend on the mode of sediment movement and are functions of the ratio of shear-velocity to fall-velocity. Only the exponent a is of importance in the final solution and the functional relationship can be expressed as:

a	$\sqrt{gyS}\,/\,w$
1/4	< 1/2
1	= 1
9/4	> 2

For flood flow conditions, the assumption can be made without serious error that $\tau_0'/\tau_c - 1 = \tau_0'/\tau_c$. At equilibrium $Q_t = Q_c + Q_o$ and $\bar{c}_1 Q_c = \bar{c}_2 Q_t$; algebraic manipulation then permits the depth ratio to be expressed as a function of the contraction parameters

5 "River Bed Scour During Floods," by E. W. Lane and W. M. Borland, Transactions ASCE, Vol. 119, 1954.

6 "The Total Sediment Load of Streams," by E. M. Laursen, Journal of the Hydraulics Division ASCE, Vol. 84, No. HY 1, February 1958.

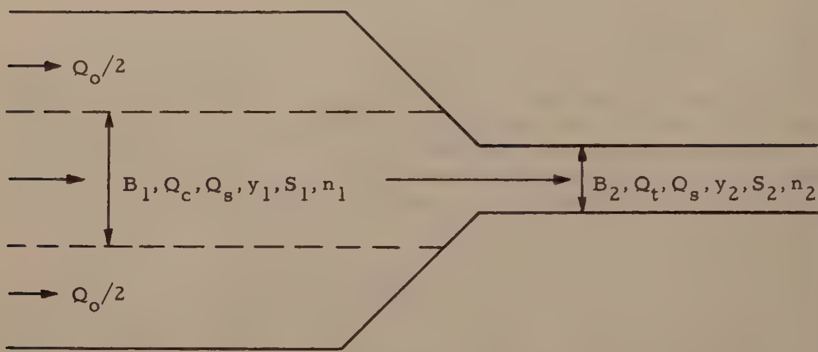

FIG. 1.—DEFINITION SKETCH OF LONG CONTRACTION

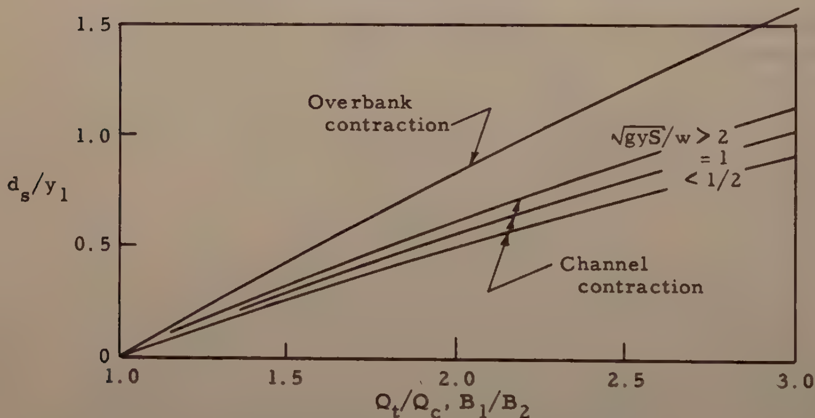

FIG. 2.—SCOUR IN A LONG CONTRACTION

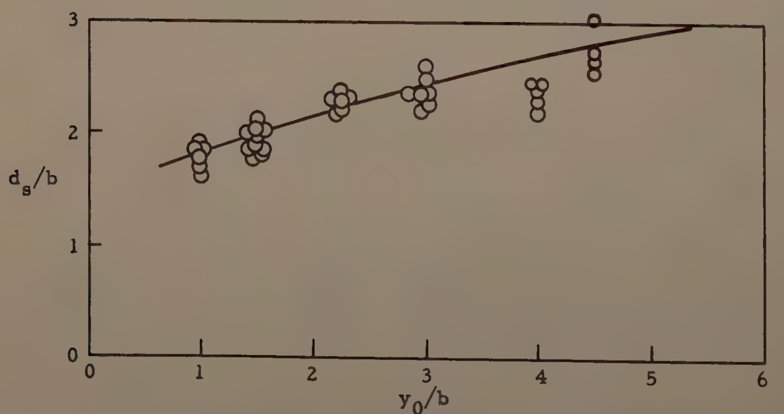

FIG. 3.—EQUILIBRIUM DEPTH OF SCOUR AT A PIER

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{\frac{6}{7}} \left(\frac{B_1}{B_2}\right)^{\frac{6}{7} \frac{2+a}{3+a}} \left(\frac{n_2}{n_1}\right)^{\frac{6}{7} \frac{a}{3+a}} \quad \ldots\ldots\ldots \quad (6)$$

Since the ratio of the n values should not be too different from unity and the power is at the most 0.37, this factor can be safely neglected. Assuming that the material scoured out during a flood is redeposited over a large area so that the depth of scour $d_s = y_2 - y_1$, the foregoing equation for an overbank construction reduces to

$$\frac{d_s}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{\frac{6}{7}} - 1 \quad \ldots\ldots\ldots \quad (7)$$

and for a channel contraction to

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.59} - 1 \text{ for } \frac{\sqrt{gyS}}{w} < \frac{1}{2} \quad \ldots\ldots\ldots \quad (8a)$$

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.64} - 1 \text{ for } \frac{\sqrt{gyS}}{w} = 1 \quad \ldots\ldots\ldots \quad (8b)$$

$$\frac{d_s}{y_1} = \left(\frac{B_1}{B_2}\right)^{0.69} - 1 \text{ for } \frac{\sqrt{gyS}}{w} > 2 \quad \ldots\ldots\ldots \quad (8c)$$

These relationships are presented graphically in Fig. 2.

The case of the long contraction is of interest not only for its own sake, but for the light its solution sheds on the nature of scour in rivers. The shear-velocity in the contracted and uncontracted reaches of a particular stream should not be very different; so that for a given mode of sediment movement the depth of scour for a channel contraction is, as a first approximation, dependent only on the geometry of the contraction as described by the width ratio and the approach depth. The assumptions which give rise to the approximation are that the n value is the same in the two reaches, and that the ratio τ_0'/τ_c is large compared to unity. Although reasonable and justifiable, these assumptions result in the neglect of any secondary effect of velocity and sediment size.

In the case of the overbank constriction, the ratio of the total discharge to the channel discharge and the depth of the approach flow are sufficient to determine the depth of scour, granted the two assumptions previously mentioned. In this case, the mode of sediment movement does not affect the depth of scour. The detailed geometry of the approach conditions will, of course, determine the flow distribution by determining the velocity and cross-sectional area of the flow in the channel and on the floodplain. However, it matters not whether the velocities are high or low, or whether the flow sections are large or small; but only what discharge ratio results therefrom.

A bridge crossing is in effect a long contraction foreshortened to such an extreme that it has only a beginning and an end. The flow at the crossing cannot be considered uniform, but the solutions for the long contraction can be modified to describe the scour at bridge piers and abutments with the use of experimentally determined coefficients.

LOCAL SCOUR AT PIERS AND ABUTMENTS

Experimental Investigation.—Following the floods of 1947 in which the State of Iowa experienced a considerable monetary loss because of the damage to or the failure of bridges, the Iowa Institute of Hydraulic Research embarked on an investigation of scour around bridge piers and abutments. For the preliminary phase of the study in which the effect of the geometry of the pier or abutment was studied, a flume 10.5 ft wide and 35 ft long was constructed. Two supply lines and two tailgates permitted the wide flume to be operated as two 5 ft flumes by the addition of a center wall. Since neither a sand elevator nor a trap was built into the flume, the time period of the runs was limited and only qualitative results could be obtained. The velocity and depth of flow of the standard run, however, resulted in general movement of the bed, and the observations which were made established the nature of the phenomenon that was occurring.

Albeit qualitatively, the importance of the length-width ratio of the piers, the angle of attack of the stream against the piers, and the length of encroachment of the abutment was established in this flume, as well as the unimportance of small details of the geometry of pier or abutment. It was also demonstrated experimentally that there was an equilibrium or limiting depth of scour. In the preliminary work, piers and abutments representative of current Iowa designs were studied. The study of geometry was continued[7] with more generalized pier forms and scour arrestors. The effect of debris was was also investigated (debris can be considered as an enlargement of the pier somewhat less than the size of the debris mass depending on the permeability of the mass).

For the second phase of the program in which the effects of velocity and depth of flow and of sediment size were to be assessed, a transport flume was constructed which was 5 ft wide and 35 ft long and equipped with a sand elevator and trap. Most of the runs were made with a representative Iowa pier set at an angle of 30° to the flow. The depth of scour was measured by an electrical scour meter which could sense the position of the sand by the difference in conductivity between the water-sand mixture and plain water. As wide a range of velocity, depth, and sediment size was utilized as the characteristics of the flume permitted. Froude numbers approaching the critical and sand jumping the trap limited the maximum value of the velocity and the minimum size of the sand, respectively. The minimum value of velocity and the maximum size of the sand were limited by the requirement that there be general movement of the sediment as bed load. Nevertheless, a sixty-fold change in the rate of sediment transport was obtained—probably the most significant indicator of the range of the tests. The depth of scour at the front of the pier is shown in Fig. 3. No systematic scatter could be detected in the

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7 "An Investigation of the Effect of Bridge-Pier Shape on the Relative Depth of Scour," by D. E. Schneible, M. S. Thesis, State University of Iowa, June 1951.

measurements—indicating that the depth of scour was uniquely determined by the geometry.

This transport flume was also utilized for a model study of a bridge pier on which field measurements were made. The measurements in the field were obtained with an adaptation of the laboratory scour meter. Comparison of the model and prototype data indicated that the depth of scour could be treated as simply another length and that the equilibrium depth of scour obtained in the field.

Measurements on a system of multiple cylinders were also made. The most significant findings of these tests were that the depth of scour did not depend on the degree of contraction (or proximity of an adjacent cylinder) until the scour holes overlapped, and that the minimum depth of scour when interference did occur was given by the solution for the long contraction.

For the third phase of the study, the first flume was modified by being lengthened 7 ft, and equipped with a sand elevator and traps. The sand was supplied to five ft on one side of the flume, simulating the river channel. Clear water was supplied to the other side of the flume which simulated the floodplain, or overbank area. The experimental program for the investigation of the scour at the abutment included a variation of the ratio of discharge of overbank and channel, the depth of flow, the position and angle of the pier, and the presence of vegetal screening along the bank line. A few experiments were also conducted on the effect of spur dikes off the end of the abutment parallel to the bank, and of screening on the floodplain in the approach to the abutment.

Design Relationships For Abutments.—The observation previously mentioned, that the local depth of scour does not depend on the degree of contraction until scour holes around neighboring obstructions overlap, suggests an extension of the solution for the long contraction to the case of local scour. For a normal sand, the width of a scour hole at right angles to the flow is about 2.75 times the depth of scour. Fig. 4 is a definition sketch of a long contraction, which it will be assumed approximates the case of the abutment. The scour in the long contraction is assumed to be a fraction 1/r of the scour at the abutment d_s. Rewritten for this special case Eq. 7 becomes

$$\frac{1}{r}\,\frac{d_s}{y_0} = \left(\frac{Q_c + Q_o}{Q_c}\right)^{6/7} - 1 \quad \ldots\ldots\ldots\ldots (9)$$

Since the depth of scour is unknown, Q_c is unknown. Therefore, the discharge Q_w over an arbitrary width w is substituted where $Q_w/w = Q_c/2.75\ d_s$. (The width w should be approximately equal to 2.75 d_s and in practice a trial-and-error procedure will be necessary). Algebraic manipulation will now result in

$$\frac{Q_o}{Q_w}\quad\frac{w}{y_0} = 2.75\,\frac{d_s}{y_0}\left[\left(\frac{1}{r}\,\frac{d_s}{y_0} + 1\right)^{7/6} - 1\right] \ldots\ldots (10)$$

in which y_0 is the average depth of flow in the width w. Eq. 10, with a value of r of 4.1, is plotted in Fig. 5, together with the experimental data for runs with the approach fill and abutment normal to the direction of flow. Various set-back distances of the abutment and clear distances between the abutment

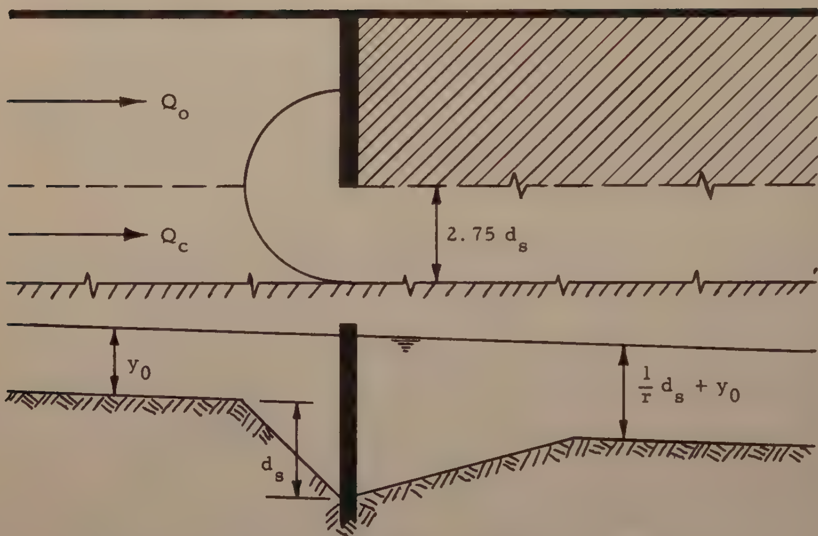

FIG. 4.—DEFINITION SKETCH OF OVERBANK CONSTRICTION

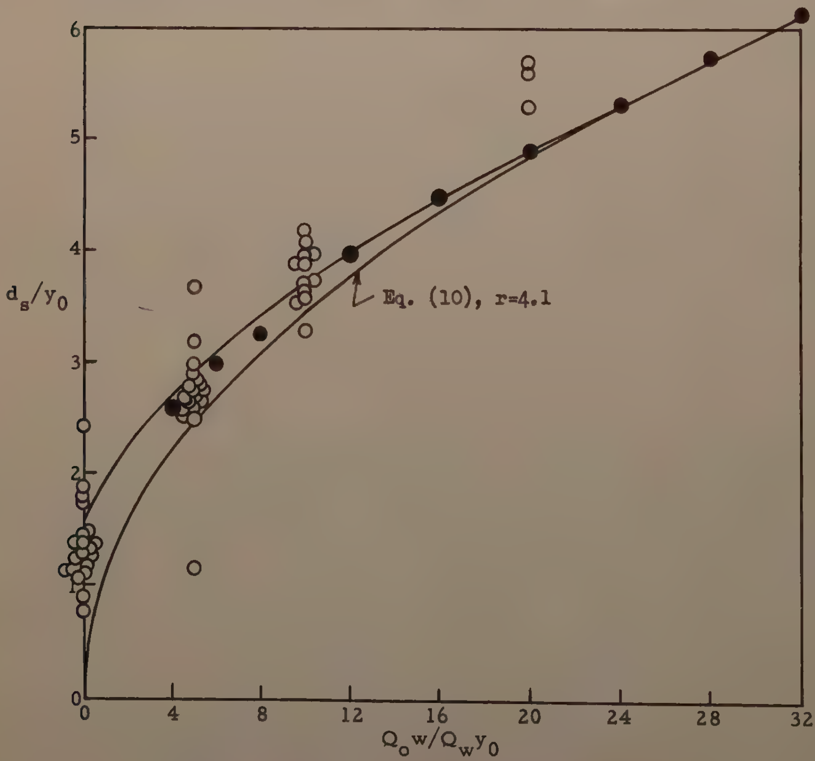

FIG. 5.—SCOUR AT AN OVERBANK CONSTRICTION

and the vegetal screen are represented among the data. The solid points are from the last, and best, series of runs in which the abutments was not set back from the bank line. The high points are from early runs in which the velocity of the approach on the overbank was so great as to be unrealistic. The floor in the overbank area was later lowered to obtain a smaller velocity of approach.

For large values of the parameter $Q_o w/Q_w y_0$, it is apparent that the approximate solution describes the scour remarkably well. For small values of the parameter, the discrepancy is due to a cross flow from the channel to the overbank in the approach to the constriction. This cross flow could be observed by means of dye streaks. Since the water-surface slope in the channel was greater than in the overbank area, the water surface at the head end of the flume was higher in the channel than in the overbank. Correcting the overbank discharge by an approximate calculation of the cross flow would shift the experimental points over to the analytic curve. This has not been done, to emphasize the importance of fully determining the overbank flow. In practice, the rate of flow on the floodplain will be determined by considering the slope of the water surface, the depth of flow and the area of the flow section, and estimating an n value. If the overbank flow is small, cross flow in the area immediately upstream of the constriction can give rise to a condition similar to that in the laboratory flume. If this cross flow is possible, but cannot be evaluated, the upper curve in Fig. 5 should be used.

A similar approximate solution can be obtained for the case of an encroaching abutment. Using Eq. 8a for the condition of bed-load movement ($\sqrt{gyS}\,/\,w < 1/2$), this relationship is

$$\frac{\ell}{y_0} = 2.75\ \frac{d_s}{y_0}\left[\left(\frac{1}{r}\ \frac{d_s}{y_0} + 1\right)^{1.70} - 1\right] \quad\ldots\ldots \quad (11)$$

where ℓ is the effective length of the abutment such that (denoting the flow obstructed by the abutment as Q_ℓ)

$$\frac{Q_\ell}{\ell\, y_0} = \frac{Q_w}{w\, y_0}$$

Eq. 11 with r equal to 11.5 is plotted in Fig. 6 together with some experimental points. The square points were obtained during an investigation of the relief bridge problem and are for the case of a vertical, blunt-ended, normal wall. The round points are data from the investigation of multiple cylinders in which there was no interference effect.

Among the variations in geometry tested was the angle of incidence between the direction of the approach fill and the direction of the flow in the channel. The results are presented in Fig. 7 as a multiplying factor, or coefficient, K_θ to be applied to the depth of scour calculated for the normal crossing ($\theta = 90°$). Although only determined for the overbank case, these coefficients should also be applicable to the encroaching abutment. A check on the points for the cylinders plotted in Fig. 6 would indicate that the effective angle of the curved wall of the cylinder is about 45°.

The effect of setting the abutment back from the normal bank of the stream is difficult to assess. In the laboratory experiments no measurable effect could be noted. The long contraction solution can be further modified to consider that a part of the overbank flow remains on the floodplain, and that the

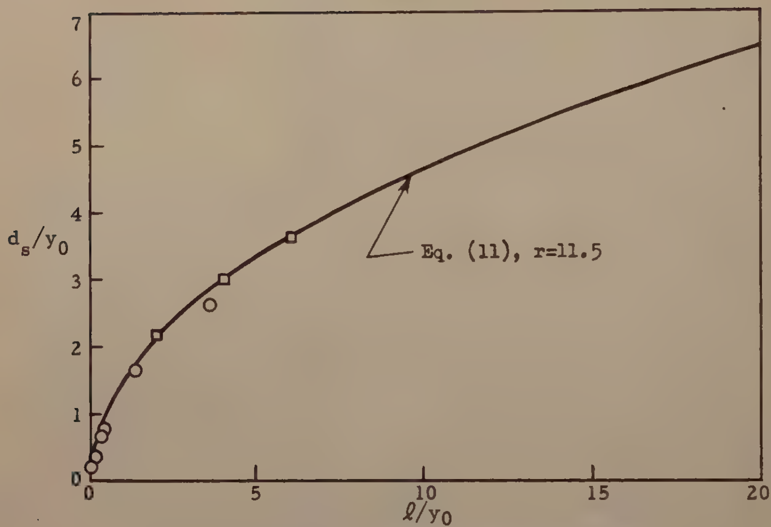

FIG. 6.—SCOUR AT AN ENCROACHING ABUTMENT

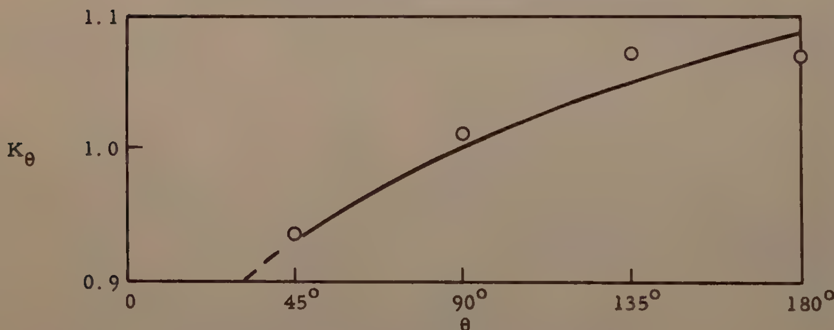

FIG. 7.—EFFECT OF ANGLE OF INCIDENCE

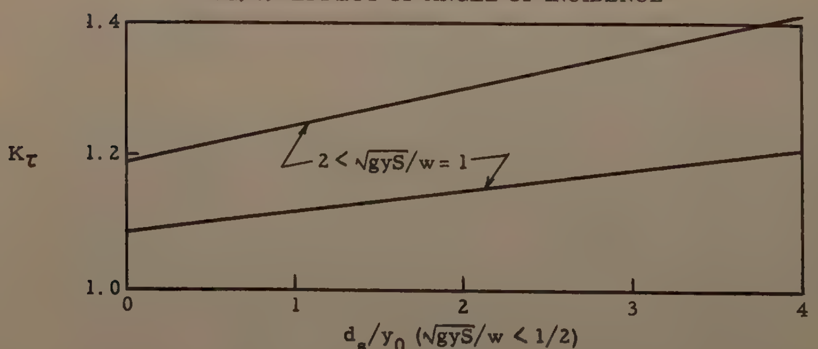

FIG. 8.—EFFECT OF THE RATIO OF SHEAR-VELOCITY TO FALL-VELOCITY

scour hole moves back with the abutment. This approximate solution also indicates that the effect of set back is small. Both the experiments and the analysis, however, do not model a real set back in certain respects. Because of the bank, the depth of flow in the vicinity of the abutment will be less. If the set back is very large, the presence of the river nearby is immaterial. The condition is then a special case of a relief bridge and the scour is due to clear water. If the bank material contains considerable clay or is well covered with a good, well-rooted, tight turf the material near the abutment might be very resistant to scour.

For the case of the encroaching abutment, the mode of sediment movement will affect the depth of scour. An approximate evaluation of this effect can be obtained by comparing Eq. 8a, b and c. For the same values of B_1/B_2 the depth of scour for the different modes of movement were calculated. Defining K_τ as the ratio of the depth of scour under suspended-load conditions to depth of scour under bed-load conditions, Fig. 8 was prepared. Note that the fall velocity should be that of the bed material being scoured out. The so-called wash load should be ignored. The effect of a change in the mode of movement should be applicable also to the local scour at a pier. Experiments made with a circular cylinder and a bed material 0.04 mm in diameter confirmed the order of magnitude of the effect. The depth of scour—very difficult to measure and extremely sensitive to unsteadiness of the flow—was about 50% greater than for bed-load conditions. The ratio of shear-velocity to fall-velocity in this experiment was about 20, and the concentration between 5 and 10% by weight, that is, 50,000 and 100,000 parts per million. The ratio between the rate of suspended-load movement and the bed-load movement was about 500 to 1.

Design Relationships For Piers.—All the available data on piers were adjusted to scour around a rectangular pier aligned with the flow, and the design curve shown in Fig. 9 was drawn with conservatism and with due regard for the reliability of the various data. Also shown in Fig. 9 in Eq. 11 with $r = 11.5$ (note that $b = 2\ell$). The most important aspect of the geometry of the pier was the angle of attack between the pier and the flow, coupled with the length-width ratio of the pier. A family of curves is shown in Fig. 10 as a multiplying factor $K_{\alpha L}$ to be applied to the depth of scour obtained from the basic curve. If the pier is set at an angle to the flow, it is recommended that no allowance be made for shape. If it is certain that the pier is aligned with the flow and will remain so during the life of the bridge, a shape factor can be applied as shown in Table I.

All the experimental work on which the design curve was based involved bed-load movement. If the mode of movement is different, the ratio of shear-velocity to fall-velocity factor K_τ in Fig. 8 should be used.

Besides the local scour at the pier, it is possible that the scour centering at the abutment may extend its area of influence out to piers near the bank. Thus, a fraction of the depth of scour at the abutment may have to be added to the local depth of scour. Typical scour holes have been shown whereby this effect can be estimated.[3]

In the bulletins of the Iowa Highway Research Board the application to design is fully discussed, including schematic design examples illustrating the use of all of the design relationships. On the basis of these relationships the prediction of local scour at piers and abutments is simple and straightforward. In order to use the relationships, however, the flow conditions at the

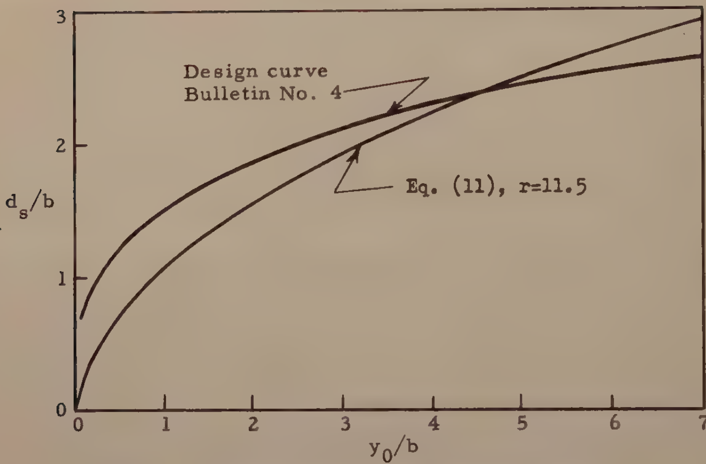

FIG. 9.—SCOUR AROUND BASIC BRIDGE PIER

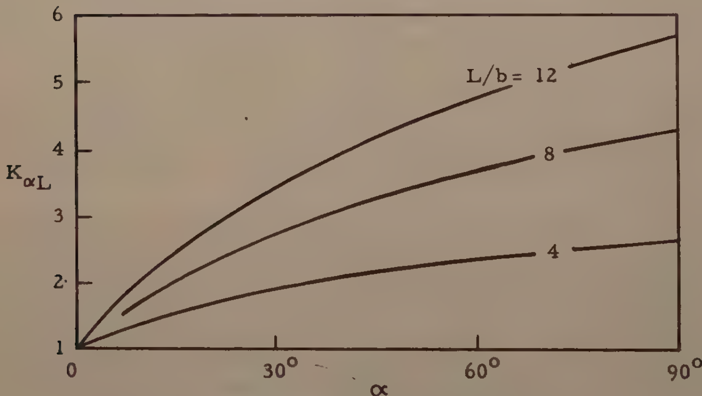

FIG. 10.—EFFECT OF ANGLE OF ATTACK

TABLE I

Shape coefficients K_s for nose forms.
(To be used only for piers aligned with flow.)

Nose form	Length-width Ratio	K_s
Rectangular		1.00
Semicircular		0.90
Elliptic	2 : 1	0.80
	3 : 1	0.75
Lenticular	2 : 1	0.80
	3 : 1	0.70

bridge crossing must be known. As any civil-hydraulic engineer knows, the forecasting of the river flow of the future is not easy. Few gaging sites have been operated for 100 years, and 100 years of record are not sufficient to define precisely the magnitude of a 100-year flood. Unfortunately, even rarer floods during the anticipated life, say 50 years, of a bridge, have a probability of occurence which is large enough to be reckoned with. This problem, of course, is simply the old basic problem of hydrology.

In addition to magnitude, other characteristics of the flow to be expected must be foreseen; the stage, the direction of flow (which may vary with stage), the division of flow between channel and overbank (which will vary with stage and the stage of the floodplain). Although these details can never be foretold with complete confidence, they are essential to the prediction of the scour which will occur. The evaluation of the flow characteristics must be judicious, and a calculated risk must be accepted; otherwise, the worst conditions must be assumed and overdesign will usually result. If sufficient control can be exercised over the valley upstream from the bridge crossing, the risk can be reduced to the chance of the flood occurrences. Each bridge crossing will have to be treated individually, and in the final analysis economic considerations will dictate the solution.

Backwater at Bridge Crossings.—When scour occurs at a bridge crossing, the flow pattern which then obtains does not resemble that of a two-dimensional constriction. The flow approaching the obstruction (pier or abutment and embankment) ducks beneath the surface and passes through the constriction as a spiral roller. The remainder of the flow passes over this roller and through the constriction barely noticing the piers and abutments, or that there is a constriction. There is no contracting jet as in a two-dimensional slot; or as in an open-channel constriction with inerodible boundaries. As a result, the velocity increases only as it must to transport the sediment load downstream from the bridge; the energy losses at and due to the bridge crossing are minimal. Since the flow is disturbed, there are, of course, some losses. Fig. 11 shows the measured rise in water surface in the abutment flume. This measured rise is partly due to the velocity-head change between the approach flow in the channel and the flow in the contracted section downstream from the bridge. The average depth in the downstream section can be obtained from Eq. 7 and the curve for the velocity-head differential has been evaluated on this basis. The backwater at the bridge is the difference between the velocity-head differential and the measured rise of the head of the flume. The backwater in a comparable inerodible flume according to Liu, Bradley, and Plate[8] is also plotted for comparison. It is readily apparent that the increase in the flow section due to the scour reduces the backwater to a fraction of the amount that otherwise would occur.

It is likely that a major cause of backwater at a bridge crossing on an alluvial stream might be the resistance to flow of the contracted stream in the reach below the bridge. Depending on bank height and cover on the floodplain the cross flow from the stream back to the floodplain might take place very slowly. If the resistance to flow is greater in the contracted reach than in the normal stream, backwater will result. Moreover, any backwater which does

8 "Backwater Effects of Bridge Piers and Abutments," H. K. Liu, J. N. Bradley, and E. O. Plate, Civil Engineering Section Project Report CER. 57HKL2, Colorado A. and M. College, Fort Collins, Colorado, 1957.

occur in an alluvial river tends to cause aggradation upstream, and a concomitant lengthening of the backwater curve.

SUGGESTIONS FOR FURTHER STUDIES

Field Measurements.—In order to apply with confidence the relationships proposed for predicting the scour at bridge piers and abutments, field measurements are needed to verify the conformity of model and prototype. Some of these measurements should be made at sites of extremely simple geometry like that of the Skunk River pier near Ames, Iowa. Such sites can be easily modeled in the laboratory, and no questions should arise as to extraneous effects. Other measurements should be made at sites of complex geometry which perhaps cannot be modeled in the laboratory. The scour can be pre-

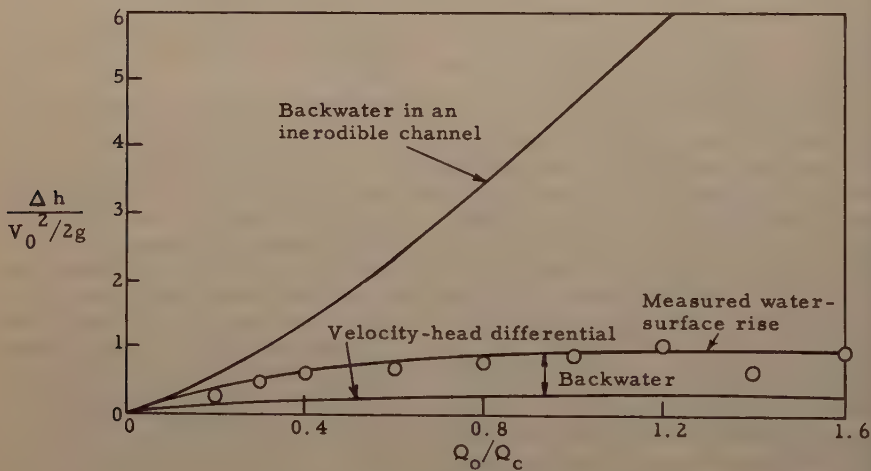

FIG. 11.—BACKWATER IN AN ALLUVIAL CHANNEL

dicted with the aid of the relationships presented, and any discrepancies may indicate factors that should receive further study.

The experience of the past can be used to substitute for field measurements. By hindcasting, the scour which occurred around piers and abutments at selected crossing during major floods can be evaluated. If bridges that failed and that did not fail during the same flood are checked, a correlation should be possible among hindcasted depth of scour, permissible depth of scour (based on foundation design), and failure or non-failure.

Laboratory Investigations.—The one general aspect of scour at bridge crossings which has not been discussed is the scour at relief bridges. Relief bridges are openings placed in the highway embankment as it crosses the floodplain of the valley. For design purposes the flow through the relief bridge must be considered to consist of clear water. If there is a sediment load it will tend to be much finer in size than the material which will be scoured out. The depth of scour due to clear water is most assuredly a function of the

velocity of flow and the sediment size as well as the geometry of the opening and its environs. The limiting condition for this type of scour is a boundary shear that is equal to the critical for the material of which the boundary is composed. Since there is no supply of sediment coming into the scour hole, at the limit there can be no sediment going out of the scour hole. This limit is finite, but is reached asymptotically in infinite time. It is very difficult in the laboratory to scale all factors controlling clear water scour and a combination of experiment and analysis is indicated.

For an abutment which is set back from the normal bank less than the depth of scour, the relationships proposed herein should be applicable. An abutment set back over three times the depth of scour should be considered a relief bridge. Following a successful investigation of the relief bridge problem, it would seem in order to study the effect of set backs between these limits.

One could, of course, go on studying the effect of various geometries of piers, abutments and overall crossing. However, the results of this investigation indicate that the effects of these details are minor, and of less importance than the error that can be expected in the evaluation of the flow conditions at the site. Further studies of geometry should probably be deferred until field measurements indicate they are of sufficient importance.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the contributions of the many others who were associated with him in this investigation of scour at bridge crossings. On the one hand there was the continued support and encouragement of Dr. Hunter Rouse, Director, Iowa Institute of Hydraulic Research; Mr. Mark Morris, Director of Highway Research, Iowa State Highway Commission; and Mr. Carl Izzard, Chief, Hydraulic Research Branch, Bureau of Public Roads; and on the other, the aid of the many research assistants who labored so long and so well, and especially Mr. Arthur Toch, Research Engineer, I.I.H.R. who, as a colleague for years, contributed in many ways, and Dr. Phillip G. Hubbard, Research Engineer, I.I.H.R., who was responsible for the development of the scour meter.

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SEDIMENT TRANSPORT AND DELTA FORMATION

By E. Kuiper,[1] F. ASCE

SYNOPSIS

This paper deals with sedimentation computations that were carried out in connection with dike design for the Saskatchewan Delta Reclamation Project. The first part of the paper gives a description of the regime of the Saskatchewan River in general and of the Saskatchewan Delta in particular. The second part presents some observations on delta formation. The third part discusses the sedimentation problems that were encountered in the design of dikes. The fourth and final part presents the method that was adopted to determine the future rate of sedimentation on the floodplains adjacent to the projected dikes.

REGIME OF SASKATCHEWAN RIVER

The Saskatchewan River system, Fig. 1, originates in the Rocky Mountains, where some of the peaks exceed an elevation of 10,000 ft. above sea level and where the permanent creeks begin to flow at an elevation of 5,000 ft. to 7,000 ft. Most of the runoff in the mountains comes from the annual snowmelt in June, which may last for several weeks. During a normal snowmelt, there is no

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

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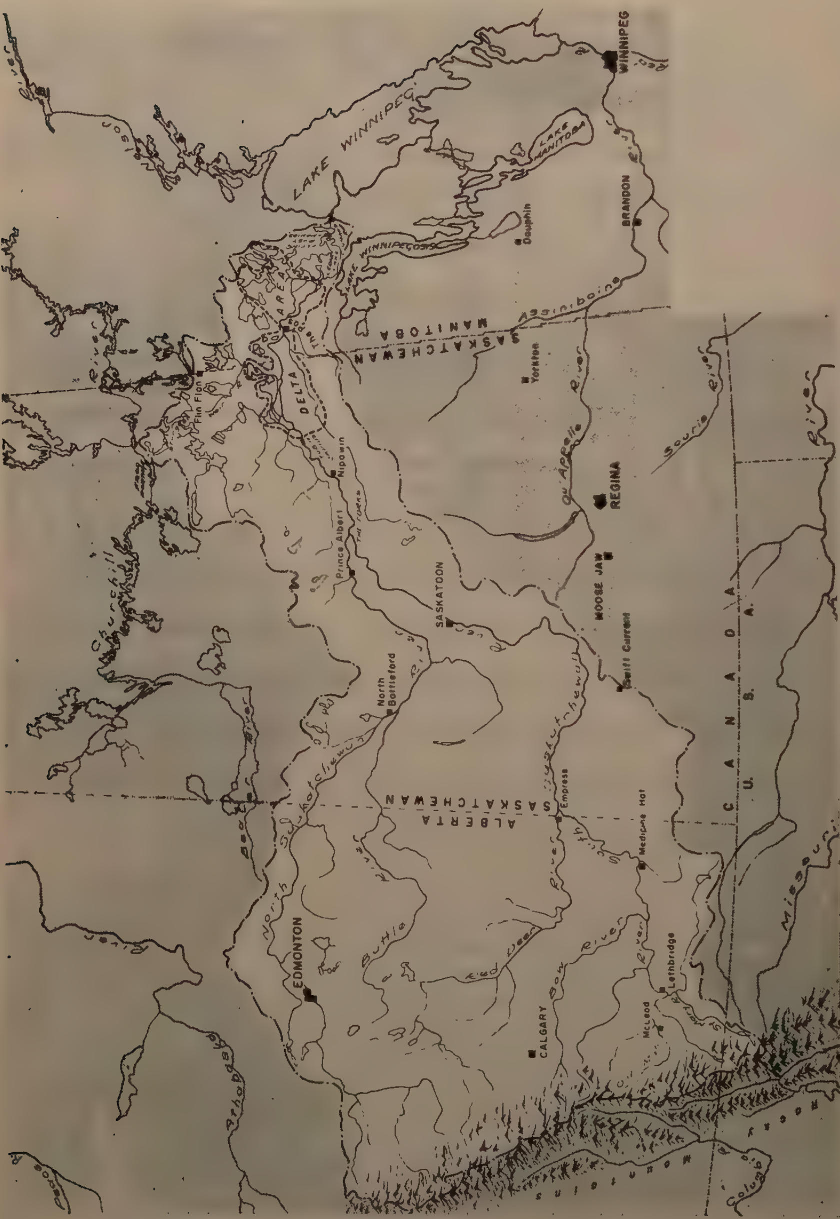


FIG. 1.—SASKATCHEWAN BASIN

sediment transport to speak of. The water in the streams is clear and the pebbles and boulders on the stream bed do not move. Most of the sediment transport takes place during short, concentrated rainstorms. At such a time, the creeks become torrents and they bring down the debris from the mountain slopes in tremendous quantities. The majority of the mountain streams flow in V-shaped valleys with a steep gradient. They cascade over rapids and waterfalls and are apparently in a state of active erosion. Some of the streams, however, flow in valleys with a mild gradient. These streams have a braided character. The valley floor is filled with gravel and small boulders and gives the impression of being a horizontal plane between the mountain sides, over which the numerous stream channels wander back and forth at will. Of course, this valley plane is not quite horizontal, but it has a longitudinal slope which is sufficient to carry discharge and sediment load.

Numerous small headwater streams leave the foothills of the mountains at an elevation of about 4,000 ft. Their bed consists of boulders and gravel; their slope is about 10 ft. per mile; and during normal snowmelt conditions, the water has a visibility from 6 ft. to 10 ft.

At this place, one would normally expect the beginning of the alluvial fans that are often found at the foot of mountain ranges. However, in the Saskatchewan Basin, it appears that the slope of the land east of the mountains is steep enough to carry the streams with their entire sediment load so that no deposition takes place. In fact, the slope is so steep that active erosion takes place. Near the foothills, the stream valleys are small, but in an easterly direction, they increase in depth and width and unite into valleys of still greater dimentions. The visibility of the water rapidly decreases from practically clear to a visibility of about four ins. over a distance of about 50 miles, indicating that sediment is derived from local river-valley erosion.

Between Macleod and Lethbridge, a distance of about 60 miles by river, the following observations were made during a normal snowmelt flood stage in June. The river valley had a width of about 1 mile, at places narrowing down to a quarter of a mile. The depth of the valley increased in downstream direction from about 75 ft. to 125 ft., and occasionally to 175 ft. The river meandered from one valley bank to another, and where it touched, huge active landslides could be seen. The valley banks consisted of shale overlain by glacial till. There were no signs of ancient alluvial fan deposits. The river had a varying width of several hundred feet and a varying maximum depth from 6 ft. to 15 ft. The bottom consisted of coarse gravel and boulders, well-sorted and well-rounded, indicating that occasional transport of this bed material takes place. While the boat drifted on the stream, the clear tinkling sound of coarse sand and fine gravel moving over the bed could be heard, the bottom of the boat acting as a soundboard. The slope of the river was about five ft. per mile. The suspended-sediment concentration was found to range from 400 ppm to 500 ppm. The low floodplain banks of the river channel were composed of layers of sand to coarse gravel, each layer well-sorted. At one place, an old oxbow was observed at a higher elevation than the present river channel, indicating that the river is in a state of degradation. Except for a few minor rapids, the river had a rather uniform maximum surface velocity of about 6 fps. This uniform condition may be due to the fact that the river valley is excavated in soft shale, the coarse bed material being transported from upstream. Thus, there is no cause for major rapids or waterfalls. The gradient of the river is probably governed on the one hand by the amount of bed-material load that comes from the mountains, and on the other hand by the rate of erosion that takes place in

the valley. It would seem that the eroding shale banks form an important source of the suspended-sediment load that is found in the river.

Between Lethbridge and Medicine Hat, the river valley is similar to the foregoing description. The river channel and river valley gradually become wider and deeper, but the bed material remains composed of coarse gravel and small boulders, and the actively eroding valley banks are still composed of shale with an overburden of glacial drift.

Between Medicine Hat and Empress, an interesting change takes place. The river banks increase in height until they are more than 400 ft. above the river level. There are no floodplains, and immediately adjacent to the river channel, the valley banks, composed of soft sandstone overlain by shale and glacial drift, rise up in weird formations that resemble badland topography. The slope of the river decreases from about 3 ft. per mile to 2 ft. per mile, and the maximum surface velocity from 4 fps to 3 fps. The maximum depth ranges from 5 ft. to 15 ft., and the width of the river from 700 ft. to 800 ft. The interesting feature of this stretch is the rather abrupt change in the composition of the bed material. Near Medicine Hat, the river bed is composed of pebbles and small boulders. Halfway between Medicine Hat and Empress, the river bed is composed of small boulders, with intermittent stretches of sand and fine gravel. Near Empress, the river bed is composed of sand, with an occasional patch of gravel or a boulder sticking out of the sand. These last observations were made by dragging a sampler over the river bottom from a drifting boat, each time over a length of several hundred feet. The type of noise that the sampler made, plus the contents of the sampler, provided a reliable indication of the composition of the river bottom.

Between Empress and The Forks, the river valley is from one to several miles wide and from one to several hundred ft. deep. There are still no floodplains to speak of. The rolling valley sides, covered with grass and brushwood, come down to the river edge. The river channel has a slope of about 1 ft. per mile and is from one to several thousand ft. wide. Within the rigid boundaries of the valley banks, the river tends to braid. Even with a small boat and during the snowmelt flood, one frequently gets stuck in shallow water. The bed material consists of rather uniform sand with a mean diameter of 0.30 mm. There are no signs that erosion is very active in this stretch of the river.

In the foregoing discussion, the course of the Old Man River and of the South Saskatchewan River was followed. The North Saskatchewan River has very similar characteristics. The Bow River carries less sediment, probably due to its milder gradient and lesser valley bank erosion. The Red Deer River has its drainage area mostly in the rolling plains east of the Rocky Mountains. Normally, the runoff from these plains is low, but during heavy rainstorms, the river may carry a considerable discharge with a relatively high sediment concentration. This sediment is partly produced by the plains and partly by the river valley, which has in its middle part a very pronounced badland topography.

Between The Forks and the delta, the gradient of the river increases to several feet per mile. Occasional rapids are found, and the bed material consists of gravel and boulders. The valley banks are from one to a few hundred feet high and are composed of glacial drift. At places, where the river channel nudges into the valley banks, huge active bank slides over the total valley height, and several thousand feet long, can be seen. At the end of this stretch, the river channel descends about 50 ft. over Tobin and Squaw Rapids and enters the Saskatchewan Delta, shown in Fig. 2.

About 10,000 yr. ago, receding glaciers allowed the Saskatchewan River to take its present course. Near its lower end, where the river entered glacial

Lake Agassiz, most of the sediment was deposited and the formation of the delta area, as it is shown at the present time, was commenced. Deposits of coarse material were confined to channel locations, while the fine silts and clays settled down farther away from the river. After the channels were extended over some distance into the lake, while possibly being split up into several branches, a situation developed whereby the river would somewhere break through its own formed banks and choose an entirely new course. In this fashion, the Saskatchewan River changed from one group of channels to another for thousands of years. Some of the oldest channels are completely covered by subsequent sedimentation. Some of the more recently abandoned channels are still plainly visible on aerial photographs and show up in the landscape as densely wooded ridges, enclosing low-lying marsh areas. The whole delta area, disregarding local ridges and depressions, is a gently sloping plain about 30 miles wide and 120 miles long, with a gradient of about 1 ft. per mile near the upper end and 1/2 ft. per mile near the lower end. The single river channel that enters the delta cone has a slope of about 1 ft. per mile and may carry during normal flood stages a peak flow of 100,000 cfs, containing 0.3% by weight of sediment. The several river channels that leave the delta and flow into Cedar Lake, one of the remainders of the ancient glacial lake, have a slope of about 1/4 ft. per mile and may carry during normal flood stages a combined peak flow of 50,000 cfs, containing 0.1% of sediment. The difference in sediment load between the upper and lower end is used in raising the delta surface. The remainder of the sediment load is used in river-mouth extension into Cedar Lake.

The average annual sediment transport into the Saskatchewan Delta is estimated at 12,000 acre-ft. About one third of this volume is sand with an average diameter of 0.20 mm., and the remainder is wash load (silt and clay). Practically all of this sediment is transported in a suspended state and is deposited somewhere in the delta. During low and medium flood stages, the flow of water and sediment remains within the natural river banks and deposition takes place in the river channel and at the mouth of the river channels. During high and extreme flood stages, the river banks are overtopped and deposition takes place throughout the entire delta area. These three modes of sediment deposition: in the river channels, on the floodplains, and in the river mouth, will be discussed in more detail in the following paragraphs.

The river channels at the apex of the delta cone have a slope of about 1 ft. per mile, which is in accordance with the observed regime slope on the lower North and South Saskatchewan Rivers, as noted earlier. The river channels near the foot of the delta cone at Cedar Lake have a slope of about 1/4 ft. per mile. This reduction in river slope corresponds with a reduction in sediment-transporting capacity. It is estimated from observations and supporting computations that the capacity to transport the washload is hardly affected, but that the capacity to transport the sand is reduced by about 80%. As a result, sand is deposited in the river channels; the coarsest fractions upstream, and the finer fractions downstream. This is reflected in the composition of the bed material, which shows a mean diameter of 0.30 mm., 0.24 mm., and 0.19 mm., respectively, near the apex, the middle, and the foot of the delta.

During high flood stages, the river banks are overtopped and a flow of water and sediment is directed towards the marshes. Immediately beyond the edge of the river bank, the coarsest sediment begins to precipitate. Soil sample analyses show that the natural levees are composed of the coarsest fraction of the suspended sediment that is carried by the river. Away from the river, the deposited sediment becomes finer and finer, until only the wash load

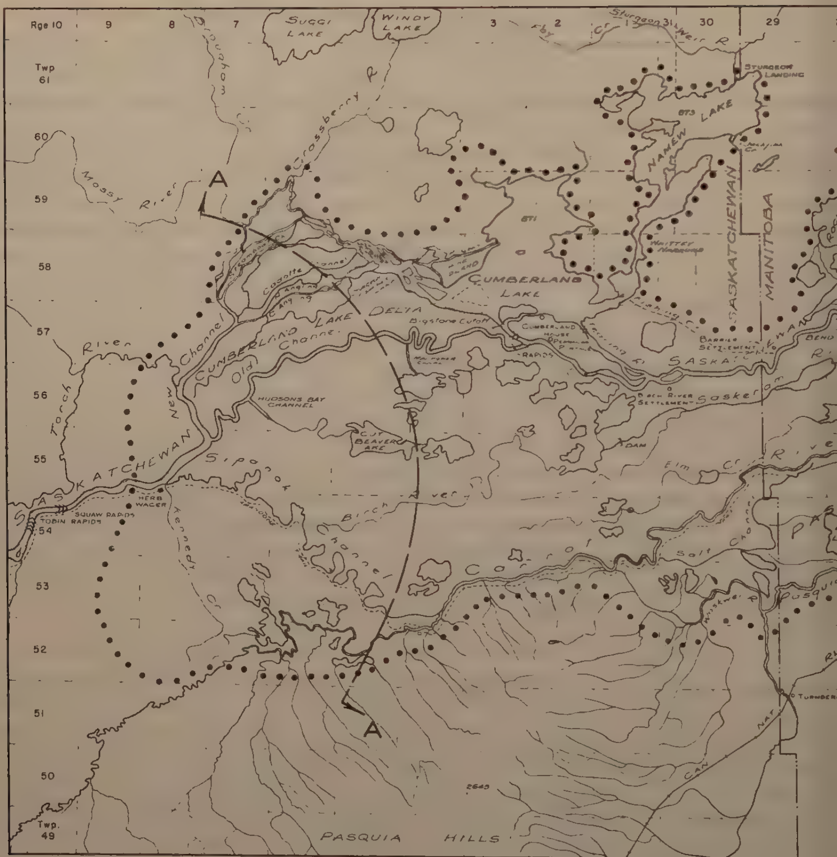
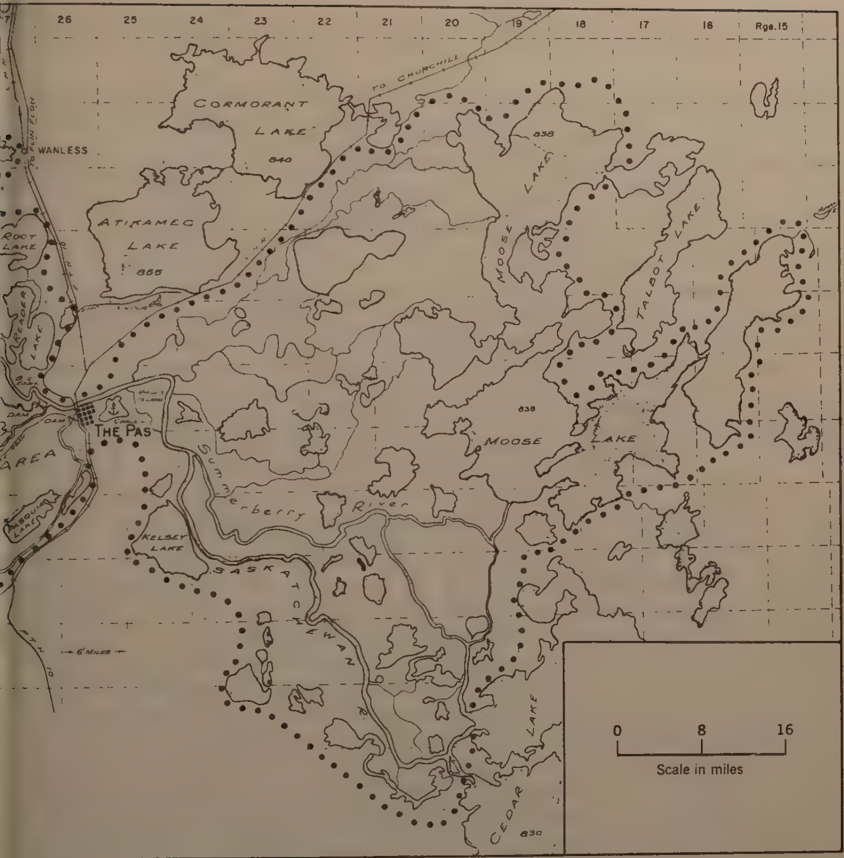


FIG. 2.—SASKATCHEWAN

remains which is deposited in the quiet marshes and lakes adjacent to the river channels. After a delay, which may range from a few days to several weeks, the overflow water, now deprived of its sediment load, returns some distance downstream via the outlet creeks of the marshes into the river channels and helps to reduce the average suspended-sediment concentration of the river flow.

At Cumberland Lake and at Cedar Lake, river channels emerge into a large body of water. Due to the inertia of the moving water, the river flows forward as a homogenous body of water into the lake and consequently, it retains its silt load for a while. After a certain distance, however, friction overcomes inertia, the water slows down, and sediment precipitates. This deposition of sediment takes place where velocities decelerate, which happens to be mostly at the flanks and in front of the retarding river current. Thus, two longitudinal bars and a number of shoals are formed. From aerial photographs, taken 25 yr. apart, it appears that the bars and shoals keep growing in elevation and that the openings in between develop into river channels. Some of the less important channels silt up until only one or a few main channels remain. All of the sand and most of the silt carried by the river is deposited near the river



DELTA

mouth. Some silt and most of the clay is deposited further away in the lake. The bottom of Cumberland Lake and the western part of Cedar Lake, for instance, are almost entirely covered with soft mud. Since Cumberland Lake is only 10 ft. deep at flood stages, occasional winds keep stirring up the bottom deposits and as a result, the water that flows out of Cumberland Lake is always discolored. Cedar Lake, however, is larger and deeper, causing all sediment to permanently settle down, so that the outlet of Cedar Lake has clear water at all times.

DELTA FORMATION

The essential process in the formation of a river delta may be summarized as follows: A river brings water and sediment into a lake or sea. Through deposition of the sediment, the river mouth is extended. Since the river needs a slope to carry its water and sediment, the lengthening of the river channel will result in a rise in river stages at any given point on the lower reach of the river. There is a limit to how much a river can rise above the adjacent land, and after this limit is reached, the river breaks through its own banks, chooses

a new course, and begins to repeat the former procedure. The manner in which this process takes place will be described in more detail in the following paragraphs.

When a river enters a lake the flow of water is not dispersed in all directions, but due to its momentum, it maintains for some distance its original direction and moves as a well-defined stream into the lake. An appreciable amount of sediment, carried by the river, will be deposited at the sides of this stream, where moving and still water meet, and will form two banks that provide further guidance to the river current. When after recession of the flood the lake and river levels drop, these banks may become exposed. This will enable them to consolidate and possibly to grow some vegetation. When the next flood comes, the same cause for further deposition on the banks is still present and may even be increased due to the vegetation, while the causes for erosion are lessened through the consolidation and the presence of the vegetative cover. When this goes on from flood to flood and no avulsion through the natural river bank takes place, the river mouth will extend like a finger in the lake. Sometimes, this form of river-mouth extension is very pronounced, and at other times, it is hardly noticeable. There are several factors which affect this phenomenon. First of all, the type of sediment carried by the river is important. A large proportion of fine sediment will produce a relatively high sediment concentration in the overflow water and hence, a rapid sedimentation of the banks. Moreover, fine sediment will result in cohesive banks that may delay the occurrence of an avulsion and thus promote the extension of a single mouth into the lake or sea. Secondly, a rapid progress of the river mouth will result in an excessive concave upward curvature of the stream profile. In other words, the slope near the mouth becomes relatively low. Hence, the mouth may protrude over a large distance into the lake or sea before the height of the river banks becomes critical for an avulsion. There are several possible causes for rapid progress of the river mouth, like a shallow lake or a large concentration of sediment. Since the slope of a graded river is determined by the bed-material load and not by the wash load, the circumstances would be ideal for finger-like extension when a river with a low concentration of bed-material load and high concentration of wash load would enter a shallow lake. The mouth would progress rapidly, the banks would be cohesive, and the slope would be small.

After an avulsion takes place, the river will be using both the old and the new channel for a certain length of time. However, there is a tendency for the old channel to silt up and to be abandoned for the following reasons. First of all, an avulsion often takes place in the outside of a river bend where the water is piled up during flood stages. This means that the new channel assumes a more or less straight course, whereas the old channel turns off to the left or to the right. Due to the tendency of the bed load to move towards the inside of a bend, the old channel will receive proportionally more sediment than water. In order to carry this sediment, the river needs a steeper slope, which it can obtain only by silting up its upper end. Even when the distribution of water and sediment over the old and the new channel is in proportion, then the new slope on the old channel still needs to be steeper because two separated channels are less efficient in transporting sediment than one combined channel.

In addition to the above causes for the actual silting up of the old channel, there is the tendency of the new channel to produce a local steep gradient at

the site of the avulsion. This is first of all because of the local difference in elevation between the river channel and adjacent land, and secondly, because the distance from the avulsion to the point where the sea or lake level is reached is usually less on the new channel, especially if deposits of the old river project far into the sea or lake. This steeper gradient in the new channel near the avulsion will cause a draw-down water profile on the river upstream. This will result in local scour and a further decrease in river stages near the entrance of the old channel, thus depriving this channel of more and more of its original flow.

The deposition of sediment in the old channel will take place in such a way that the remaining cross section is in equilibrium, as discussed previously. That means that deposition will not only take place on the bottom of the channel but also at the sides. The channel will become shallower and narrower. Finally, the entrance of the channel may become so narrow that it plugs up with debris or it may become so high that it becomes grown over with vegetation during low-water periods. After this has occurred, the channel will be abandoned and be filled up with marsh vegetation.

The rapidity of this silting-up process depends upon the proportion of excess bed-material load that enters the channel. If this is low, for instance, due to the entrance being located near the convex side of the main current, the river will silt up slowly. In the meantime, the new channel also silts up due to normal aggradation. Hence, the initial degradation upstream of the avulsion is checked and is reversed to aggradation. It may even be that the aggradation of the new channel catches up with the aggradation of the old channel and that both rivers become equal partners. The aspect that these two rivers are less efficient in their sediment transport than the former one river channel causes their slopes to be steeper. Consequently, the river channel upstream of the avulsion will aggrade above the level that it had at the time of the avulsion. After this has continued for awhile, another avulsion may take place, eliminating the two old channels and replacing them by one new channel.

The formation and aggradation of the new channel, which initially flows through marshes and shallow lakes adjacent to the old channel, is somewhat similar to the formation of the old channel, as discussed in the above. However, one point of difference is of interest. In the old channel, it was assumed that the river bed consisted of normal river bed-material. The new channel, flowing through low areas where formerly fine sediment was deposited, may find a bottom of cohesive material that resists scour. Hence, the formation of the cross section may be different from what it would be if the river flowed over its own bed-material deposits. It could be expected that in the beginning the channel will be relatively wide. After the river digs into the clay deposits part way, it could be expected that normal meandering or casual shifting will be less pronounced. It could also be expected that the river slope can be steeper than would be in accordance with the transported bed-material load, due to its resistance against scour.

From the foregoing discussion, it follows that an avulsion may lead to the presence of two more or less stable and more or less active river branches. If this is repeated a number of times, the familiar picture of river arms in a delta, comparable to the veins in a leaf, is obtained. In downstream direction, the river channel will become increasingly smaller due to repeated bifurcating.

There is another reason why river channels in a delta decrease in size in downstream direction. During flood stages, when the river carries most of its sediment and when the formation of the channels takes place, water consistently overflows the river banks towards the adjacent marshes and lakes. As a result, the flow that remains in the channel will become increasingly smaller in downstream direction. Since a channel is formed in accordance with the volume of its flow, it follows that the channel capacity will also decrease in downstream direction.

The consequence of this decrease in channel capacity is that only one flood flow with a certain magnitude will cause uniform overflow over the delta. Floods of greater magnitude, which occur less frequently but which have a higher concentration of sediment and are of a longer duration, will flood more severely the upper part of the delta area; while floods of relatively small magnitude will overflow more extensively in the lower part of the delta area. In this way, the sedimentation is kept in balance.

SEDIMENTATION PROBLEMS

It was found from soil surveys that practically all the alluvial deposits in the Saskatchewan Delta, covering an area of about 2,000,000 acres, are suitable for agricultural development provided the water table can be controlled. An area of 150,000 acres, near The Pas, has recently been reclaimed, while plans are being made for the future reclamation of an additional 1,000,000 acres of delta land.

Reclamation of part of the delta surface will result in a smaller area being available for deposition of sediment and hence, in a faster rise of the surface level. This in turn will increase the flood elevations and thus endanger the projected dikes. The problem arises to determine the future sedimentation of the floodplain in quantitive terms and to find possible remedial measures.

Generally speaking, there are three principal methods of coping with the present sedimentation problem. The first method is by reduction of the sediment load through soil conservation. The second method would aim at increasing the sediment-transporting capacity of the river channel by river-training works. The third method includes all plans for controlled storage of sediment by reservoirs, or in the delta area. In the following paragraphs, the applicability of these methods will be discussed.

In cases where the sediment, carried by the river, finds it origin in surface erosion of the drainage basin, soil conservation methods could be effective. On the one hand, they improve the usefulness of the land, and on the other hand, they may provide a solution to the sediment problem. However, the sediment carried by the Saskatchewan River finds its origin not only on the prairie lands, but also in stream-valley erosion in the upper part of the watershed. To reduce this type of sediment supply would call for extensive channel-protection works. Without entering into detail, it can be stated that the benefits in the delta area would be too small to justify the cost of such works.

The method of increasing the sediment-transporting capacity by river-training works is applied extensively on rivers in Europe and occasionally on rivers in North America. In most cases, these works also serve the purpose of maintaining a navigation channel and protecting valuable shoreline property. In the Saskatchewan Delta, such works, which may cost anywhere from $50,000 to $500,000 per mile of river channel, would be too costly to be justified by the benefits, at least for the time being.

Hence, if the sediment cannot be retained at the place of its origin, nor carried through to the mouth of the river, then it must be deposited somewhere along the course of the river, and the problem is to find the location that will do the least damage. Storage in an upstream sediment-control reservoir would provide an adequate solution. For the first few hundred years, the sedimentation problem in the Saskatchewan Delta will not be so severe as to justify the construction of an exclusive sediment-control reservoir. However, if upstream reservoirs are going to be built for other purposes, the feature of sediment control will be added automatically. It could even be expected that the consequent reduction in sediment load will cause degradation of the river channels in the delta, and thenceforth, will lower flood stages and improve drainage.

If no upstream reservoirs are going to be built during the first few hundred years, the Saskatchewan River will deposit its sediment load in the delta. At the present time, the larger part of the sediment load is deposited in the Cumberland Lake Delta. It was from a quantitative sedimentation study that this situation can be maintained for another 150 yr. From then on, increasing quantities of sediment will pass the Cumberland Lake Delta and will be deposited in stream channels and on floodplains downstream. For some time to come, the only way to cope with the consequent rise in flood levels will be to raise the adjacent dikes accordingly.

There is undoubtedly a limit to the possibility of raising dikes to keep pace with the natural sedimentation outside of the dikes. The ultimate slope of the river whereby it can carry the complete sediment load throughout the delta area is about 1 ft. per mile. A line under this slope, beginning at the future mouth of the river near the outlet of Cedar Lake, will reach a point near Cumberland House more than 100 ft. above its present elevation. It appears, therefore, that the area which is presently going to be reclaimed will have to be sacrificed sometime in the far future. However, it would seem entirely feasible to compensate for this loss by reclaiming the areas that have been silted up in the meantime. Thus, a situation would develop whereby every thousand years or so, another section of the delta area is made available for reclamation. When spread over periods of a thousand years, the cost of moving the inhabitants from the area, which is going back to the river system into the area that is going to be reclaimed, appears very small and should be no objection to present reclamation.

After having decided that the most appropriate remedial measure to cope with future sedimentation of the floodplain would be to raise the dikes as needed, the remaining problem was to determine the rate of sedimentation adjacent to the dikes.

SEDIMENTATION COMPUTATIONS

A serious attempt was made to compute in detail how the future deposition of sediment in the Saskatchewan Delta would take place. Precise sediment sampling was carried out at a dozen stations. Analytical computations were applied to determine the existing mode of transport and to estimate the rate of deposition of all sediment fractions for new equilibrium conditions. However, in the end, so many assumptions had to be made that the reliability of the answer became highly problematical, particularly because it was found that slight changes in the assumptions, well within reason, could produce significant changes in the end result. It was, therefore, decided to determine the rate of future sedimentation, not on the basis of precise equilibrium calculation, but

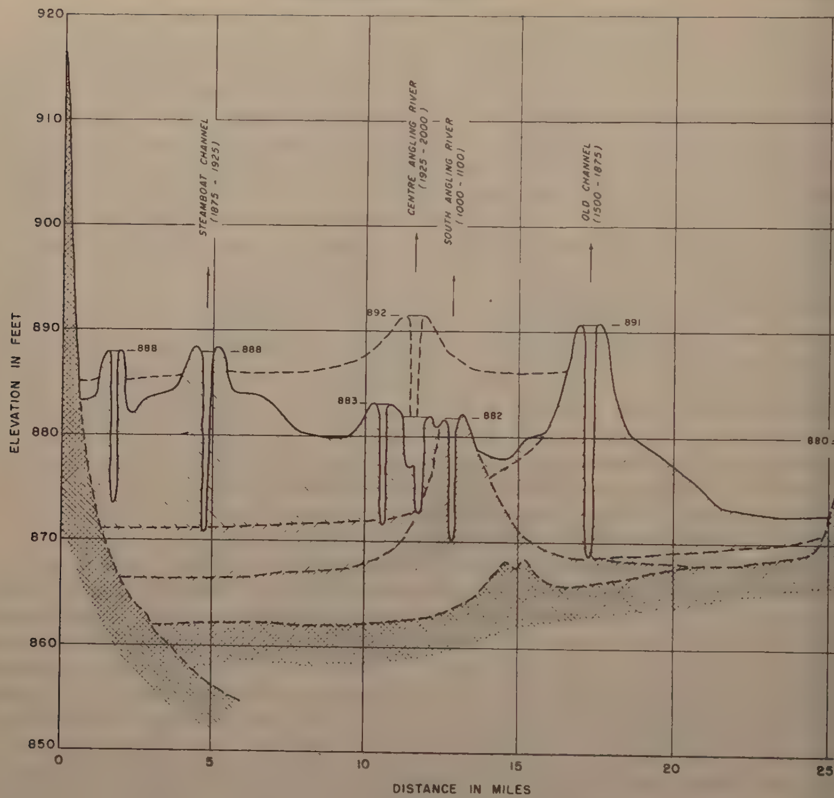
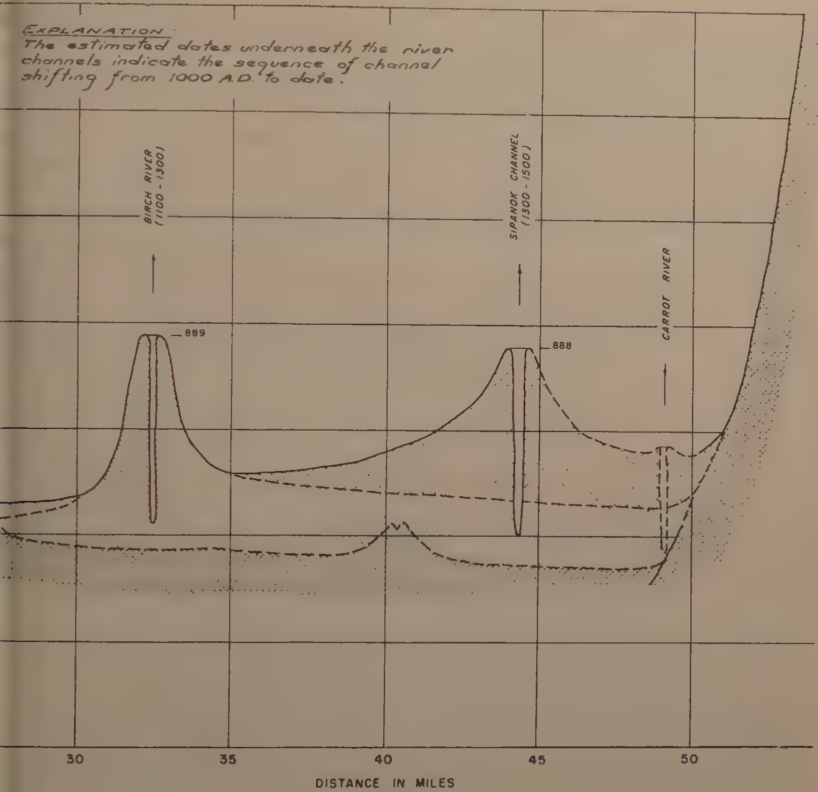


FIG. 3.—CROSS SECTION A-A

on the broad basis of an estimated average annual sediment load coming into the delta and an estimated distribution of deposition over the delta surface. Before these two items are discussed in some detail in the following paragraphs, it may be appropriate to point out at this place that detailed analysis and precise calculations were definitely helpful in exploring the nature of the sediment problem, and providing insight into the proper solution of the problem. However, the present sediment problems appeared too complicated to insist on precision to the very last answer.

The average annual sediment load was estimated on the basis of sediment sampling carried out during 2 yrs. A relationship was established between river flow and total sediment transport. From this relationship and the flow duration curve, a sediment-transport duration curve was prepared, which in turn yielded the average annual sediment-transport figure of 8,000 acre-ft. per yr. Since the 2 yr. of observation were preceded by about 10 yr. of above-normal rainfall conditions over the drainage basin, resulting in a better-than-normal vegetative cover over the sediment-producing prairies, it was decided to increase this figure by 50%, resulting in a total average annual sediment transport of 12,000 acre-ft. per yr.

The average distribution of sediment deposition over the delta area was estimated from a study of profiles and cross sections over the delta cone. One

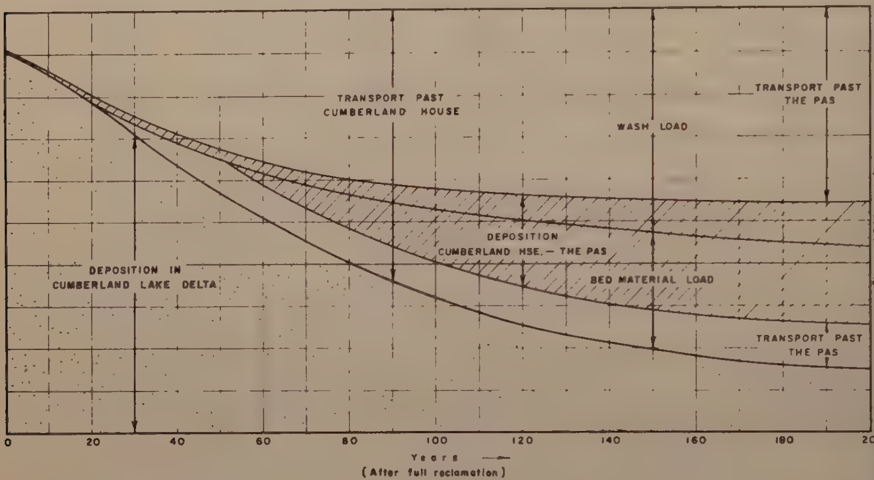


OF SASKATCHEWAN DELTA

such a cross section is shown on Fig. 3. Topographic surveys were carried out to determine the elevation of river banks and marsh lands. Hydrometric surveys were carried out to determine the distribution of flow during low, normal, and flood stages of the river system. Sediment sampling was carried out at various locations in the delta to determine where sediment was deposited for certain flow conditions. Historical information was collected from journals of early explorers and from the memory of older residents.

Based on the foregoing studies, the following conclusions were reached: At the present time, about 2,500 acre-ft. per year. of sediment load is carried by the Old Channel and remains in the river channel well past The Pas. The remaining 9,500 acre-ft. is carried by the New Channel and is nearly all deposited in the Cumberland Lake Delta. Only 1,000 acre-ft., mostly wash load, leaves Cumberland Lake and is carried to Cedar Lake. Full development of the delta area includes reclamation of all land south of the Old Channel and Saskatchewan River, and will divert the Old Channel into the New Channel. By then, the New Channel will carry the full 12,000 acre-ft. of sediment load. Twenty years later, the bed-material deposits will have reached the outlet of Cumberland Lake and increasing quantities of bed material will begin to flow down the Saskatchewan River. Thirty years later, the Saskatchewan River, between Cumberland House and The Pas, will carry its full capacity of bed material load and deposition will begin afterwards, starting at Cumberland House

and gradually extending in downstream direction. The foregoing sedimentation is graphically illustrated in Fig. 4. It was estimated that 100 yr. from now the extreme flood levels in the Cumberland House would be raised by about 2 ft., in The Pas area by 1/2 ft., and near the apex of the delta area by a negligible



EXPLANATION
Old channel will be closed off Hence all flows through Cumberland L Delta
From 0-20 years Washload past Cumberland increases from about 10 to 20% of total. A fraction of this is deposited between Cumberland House and The Pas Bed material has not reached Bigstone Cut-Off.
From 20 to 50 years: Washload past Cumberland House increases from about 20 to 40% of total. Bed material is now passing Cumberland House at an increasing rate, but can all be carried by the river between Cumberland House and The Pas.
From 50 years on Washload past Cumberland House steadily increases A fraction increasing from 10 to 20% is deposited on the flood plains between Cumberland House and The Pas Bed material load increases beyond capacity of the river. The balance is deposited in river bed and on river banks.

FIG. 4.—DIAGRAM OF FUTURE SEDIMENT TRANSPORT FOR SASKATCHEWAN DELTA

amount. From then on, the extreme flood levels throughout the whole delta area upstream of The Pas will continue to rise from 1 to 2 ft. per 100 yr. Temporary and local deviations from this average rise may occur due to abnormal climatic conditions or due to avulsions of the river channels.

DIVISION ACTIVITIES
AIR TRANSPORT DIVISION
Proceedings of the American Society of Civil Engineers

TRAP EFFICIENCY OF RESERVOIRS, DEBRIS BASINS, AND DEBRIS DAMS[a]

By Charles M. Moore,[1] M. ASCE, Walter J. Wood,[2] F. ASCE
and Graham W. Renfro[3]

SYNOPSIS

An attempt has been made to review and summarize available information pertaining to trap efficiency and to make it available to engineers and the general public in a unified form. As additional data are collected on these and other projects, it is hoped that it will be added to that presented herein so that the subject of trap efficiency may be advanced as much as its importance warrants.

INTRODUCTION

Trap efficiency of reservoirs is a subject on which there is a lack of readily available information for use by engineers in the design of water supply reservoirs, flood control structures, and other kinds of impoundment or diversion structures. Trap efficiency is expressed as the ratio between sediment accumulation and sediment outflow. One objective of the Task Force on Reservoir Sedimentation, ASCE, has been to make a summary review of the knowledge

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

a Presented at the October 1957 ASCE Convention in New York, N. Y.

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now accumulated, but not available in a unified form to the profession at large. The authors of this paper have attempted to do this.

It appears that the most authentic information available in print is a paper by Gunnar M. Brune.[4] This paper covers the normal ponded reservoirs very thoroughly as of this date. A summary of the paper is made herein, including the more important findings and results. In addition, a brief review is made of the cooperative studies of trap efficiencies of upstream floodwater retarding structures currently being made cooperatively by the U. S. Geological Survey and the U. S. Soil Conservation Service; and of the work being done by the Los Angeles County Flood Control District. There may be other published information of which the authors are not aware.

SUMMARY OF PAPER BY BRUNE

As pointed out by Brune, the trap efficiency of a reservoir depends upon a number of factors. Among these are the ratio between the storage capacity

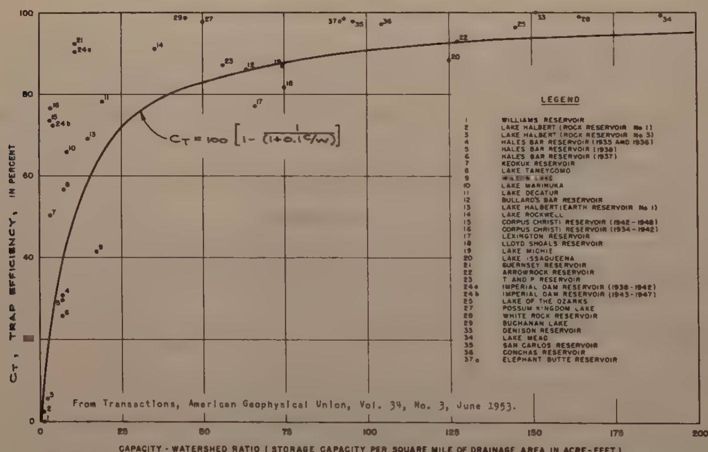

FIG. 1.—TRAP EFFICIENCY AS RELATED TO CAPACITY-WATERSHED RATIO

and inflow, age of the reservoir, shape of the reservoir basin, the type of outlets and method of operation, the size grading of the sediment, and the behavior of the finer sediment fractions under various conditions. A number of attempts have been made to correlate trap efficiency with one or more of these factors.

One of the earlier studies of trap efficiency was made by Brune and R. E. Allen,[5] who developed a curve relating percentage of eroded soil caught in a reservoir with capacity per square mile of drainage area. The trap-efficiency values given by this method, however, are necessarily low because they are

4 "Trap Efficiency of Reservoirs," by G. M. Brune, Transactions, American Geophysical Union, Vol. 34, Number 3, June 1953.

5 "A Consideration of Factors Influencing Reservoir Sedimentation in the Ohio Valley Region," by G. M. Brune and R. E. Allen, Trans., Amer. Geophys. Union, V. 22, pp. 649-655, 1941.

based upon rates of erosion which, as pointed out by L. M. Glymph, Jr.,[6] are higher than rates of sediment production to the reservoir. Glymph found that the rate of sediment production is inversely proportional to drainage area and is usually lower than the rate of erosion because of the deposition of some of the eroded material such as colluvium, on flood plains and in stream channels.

The ratio between storage capacity and inflow has been expressed in a general way by the capacity-watershed (C/W) ratio. C. B. Brown[7] first developed a curve relating C/W and true trap efficiency. The curve shown in Fig. 1, with some additional records, is represented by

$$C_T = 100\ [1 - 1/(1 + 0.1\ C/W)]$$

where C_T is the reservoir trap efficiency in percentage, and C/W is the reservoir capacity in acre feet per square mile of drainage area. There is a considerable spread in points, as may be noticed.

The main reason for the spread of points in the curve is that reservoirs having the same C/W ratio may have very different capacity-inflow ratios, as previously stated by L. C. Gottschalk:[8]

> "A reservoir in an arid or semi-arid region may have a low capacity-watershed ratio, yet not receive enough inflow in any one year to cause water to be discharged over the spillway. In contrast, the volume of mean annual flow from a watershed of equal size in a humid area may be equivalent to 25 times that of a reservoir having the same capacity-watershed ratio. In the dried region 100% of the incoming sediment load is trapped, whereas in a humid area possibly only 70% is trapped."

Allen Hazen[9] first introduced the storage, or capacity-inflow, ratio in 1914. He used it for determining reservoir requirements, however, and not as an index of sediment trap efficiency.

For individual reservoirs, curves can be drawn correlating trap efficiency with detention time in days. Whitney L. Borland has prepared[10] such a curve for Imperial Dam Reservoir, shown in Fig. 2. Such curves are quite satisfactory for specific reservoirs, since other factors such as sediment characteristics, shape of reservoir, and method of operation tend to remain constant. Detention time may be defined as the period that the inflow remains in the reservoir before being released downstream.

M. A. Churchill[11] took into account both detention time and velocity of flow through the reservoir. He has developed a "sedimentation index" which repre-

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6 "Relation of Sedimentation to Accelerated Erosion in the Missouri River Basin," by L. M. Glymph Jr., SCS-TP-102, U. S. Soil Conservation Service, Lincoln, Nebr., July 1951.

7 Brown, C. B., Discussion of "Sedimentation in Reservoirs" by B. J. Witzig, Proc., Amer. Soc. Civ. Eng., V. 69, No. 6, pp. 793-815, 1493-1499, 1943.

8 "Analysis and Use of Reservoir Sedimentation Data," by L. C. Gottschalk, pp. 131-141, Proc., Federal Inter-Agency Sedimentation Conference, Washington, D. C., Jan. 1948.

9 "Storage to be Provided in Impounding Reservoirs for Municipal Water Supply," by Allen Hazen, Trans., Amer. Soc. Civ. Eng., V. 77, pp. 1539-1640, 1914.

10 Unpublished data in files of Bureau of Reclamation, compiled by Whitney L. Borland, Denver, Colo., 1951.

11 Churchill, M. A., Discussion of "Analysis and Use of Reservoir Sedimentation Data" by L. C. Gottschalk, pp. 139-140, Proc., Federal Inter-Agency Sedimentation Conference, Washington, D. C., Jan. 1948.

sents the period of detention divided by mean velocity. His curve, relating trap efficiency to the sedimentation index for several TVA reservoirs, is shown in Brune's paper. While this curve is very satisfactory where the data are available, such information as period of detention and mean velocity is not readily available for most reservoirs.

In Brune's study a thorough search was made for all reliable records of reservoir trap efficiency, canvassing all agencies of the Federal Inter-Agency River Basin Committee known to be interested in the subject. Information was also gathered on capacity, annual inflow, shape of reservoir basin, type of outlets and method of operation, observed gravity underflows or density" currents, and any other pertinent data.

Some forty-four records were gathered and are summarized in Table 1, included herein. Of these, forty are for normal ponded reservoirs, with information on the effect of sluicing and venting operations on three. Two records are for desilting basins, and two for semi-dry reservoirs.

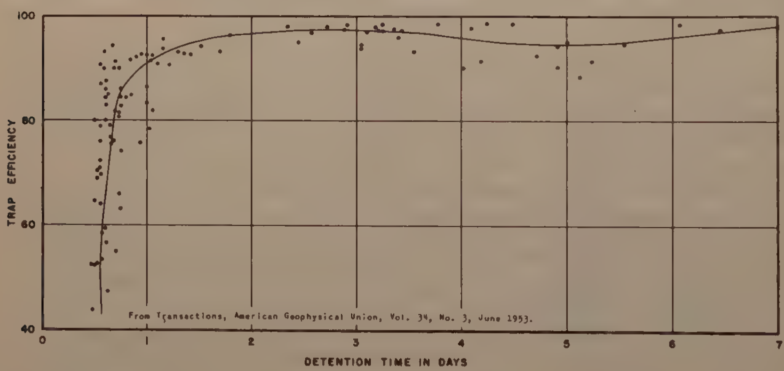

FIG. 2.—TRAP EFFICIENCY AS RELATED TO DETENTION TIME, IMPERIAL DAM RESERVOIR, YUMA, ARIZONA

As pointed out by Brune, it is possible to study trap efficiency of reservoirs by a number of different methods, and he found it necessary to use nearly all of them. Those who are interested in the details are referred to Brune's paper.

In taking suspended-load measurements downstream from a reservoir it is necessary to take them as near the dam as possible, because downstream from reservoirs streams tend to pick up a new sediment load quickly by scouring and degrading their channels. For example, as C. B. Brown has pointed out,[12] the average sediment concentration in the Colorado River below Hoover Dam as of 1947 increases from 0.006% 12.8 miles below the dam to 0.091% 98.6 miles below the dam, or 15 times. Such a change can cause grave errors in computations of trap efficiency if suspended-load measurements far downstream from the dam are used.

In computing trap efficiency it is also necessary to take dredging into account. Since dredged areas in a reservoir usually fill with sediment again in

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12 "Sediment Transportation," by C. B. Brown, Chapter XII of "Engineering Hydraulics," by Hunter Rouse, John Wiley and Sons, New York, N. Y., Aug. 1950b.

TABLE 1.—REPRESENTATIVE RECORDS OF RESERVOIR TRAP EFFICIENCY

Record	Reservoir	Stream	Location	Drainage area, in square miles	Period of record, in years	Average annual sediment production per square mile of drainage area, in tons	C/W [redacted] acre [redacted] per square mile	Average annual inflow, in acre feet per square mile	C/I, in acre feet of capacity per acre foot of annual inflow	Trap efficiency, in %	Reference, in Brune's paper
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			Normal Ponded Reservoirs								
1	Williams	E. Fork White R.	Williams, Ind.	4,700	9.3	434	1.19	747	0.0016	0	
2	Lake Halbert (Rock Res. 1)	Elm Creek	Corsicana, Tex.	5.37	69	8480	1.16	399	0.0029	2.3	(20)(4)
3	Lake Halbert (Rock Res. 3)	Elm Creek	Corsicana, Tex.	1.52	69	8480	1.63	399	0.0041	5.8	(20)(4)
4	Hales Bar (1935-1936)	Tennessee R.	Chattanooga, Tenn.	21,790	2.00	306	6.46	1260	0.0051	30.5	(18)(14)
5	Hales Bar (1938)	Tennessee R.	Chattanooga, Tenn.	21,790	1.00	306	6.39	1260	0.0051	29.7	(18)(14)
6	Hales Bar (1937)	Tennessee R.	Chattanooga, Tenn.	21,790	1.00	306	6.41	1260	0.0051	25.7	(18)(14)
7	Keokuk	Mississippi R.	Keokuk, Iowa	119,000	23	219	3.43	370	0.0093	50.0	(10)(12)(5)
8	Lake Taneycomo	White R.	Forsyth, Mo.	4,610	22.4	494	7.34	783	0.0094	56.3	(22)
9	Wilson Lake	Tennessee R.	Florence, Ala.	30,750	2.25	242	17.3	1190	0.0145	44.9	(18)(4)(14)
10	Lake Marinuka	Beaver Creek	Galesville, Wis.	138.6	72	272	8.5	549	0.0155	65.4	(10)(11)
11	Lake Decatur	Sangamon R.	Decatur, Ill.	906	24.2	341	18.9	560	0.0338	78.0	(8)(10)(6)(5)(11)
12	Bullard's Bar	No. Fk. Yuba R.	No. San Juan, Calif.	480	19.2	504	62.9	1660	0.0378	83.4	(9)
13	Lake Halbert (Earth Res. 1)	Elm Creek	Corsicana, Tex.	1.13	69	8480	15.1	399	0.0378	69.3	(20)(4)
14	Lake Rockwell	Cuyahoga R.	Kent, Ohio	205.5	36	117	34.5	698	0.0494	85.8	(11)
15	Corpus Christi (1942-1948)	Nueces R.	Corpus Christi, Tex.	16,800	6.0	65.0	2.48	45.8	0.0541	73.7	(7)
16	Corpus Christi (1934-1942)	Nueces R.	Corpus Christi, Tex.	16,800	7.6	65.0	2.92	45.8	0.0638	76.7	(7)
17	Lexington	Leonard's Creek	Lexington, N.C.	6.83	4.7	851	66.1	907	0.0730	77.2	(4)
18	Lloyd Shoals	Ocmulgee R.	Jackson, Ga.	1,414	24.3	653	74.6	923	0.0807	81.4	(26)(2)(13)
19	Lake Michie	Flat River	Durham, N. C.	167.5	8.75	408	74.5	747	0.0998	86.3	(25)(2)(13)
20	Lake Issaqueena	Six-Mile Creek	Clemson, S. C.	14.02	11.4	1569	125	987	0.127	94.2[84.1][a]	(40)(27)(4)
21	Guernsey	North Platte R.	Guernsey, Wyo.	16,200	20	292	3.80	74.2	0.0513	92.2	(29)(23)
22	Arrowrock	Boise River	Boise, Idaho	2,170	32.64	475	127	742	0.171	93.0[90.3][a]	(28)
23	T and P	Town Creek	Weatherford, Tex.	6.24	8.5	1217	55.9	293	0.191	87.0	
24	Hiwassee	Hiwassee R.	Murphy, N. C.	968	0.75	503	453	1660	0.273	98.1	(18)(14)
24a	Imperial Dam (1938-1942)	Colorado R.	Yuma, Ariz.	184,600	5.0	2170	11.7	55.5	0.211	90.2	(4)(23)(1)
24b	Imperial Dam (1943-1947)	Colorado R.	Yuma, Ariz.	184,600	5.0	2170	3.81	55.5	0.0686	72.3	(4)(23)(1)
25	Lake of the Ozarks	Osage R.	Bagnell, Mo.	14,000	17	580	145	498	0.292	96.7	(33)
26	Pardee	Mokelumne R.	Buena Vista, Calif.	430	14	217	488	1560	0.313	95.0	(9)
27	Possum Kingdom	Brazos R.	Palo Pinto, Tex.	14,098	7.75	650	49.9	63.4	0.787	98.0	(21)
28	White Rock	White Rock Creek	Dallas, Tex.	99.1	25	2104	74.6	202	0.812	99.3	(24)(2)(13)
29	Buchanan Lake	Colorado R.	Burnet, Tex.	21,000	7.1	216	44.7	53.3	0.837	98.6	(4)
30	Norris	Clinch R.	Norris, Tenn.	2,912	0.75	422	918	970	0.946	99.1	(18)(4)(14)
31	Senecaville (1939-1945)	Wills Creek	Senecaville, Ohio	121	5.1	1026	727	768	0.947	94.3	(37)(10)(6)(5)(11)
32	H. Lage Pond	Small Branch	Aspinwall, Iowa	0.038	11.0	6270	318	261	1.22	100.0	(17)
33	Denison	Red River	Denison, Tex.	38,291	6.2	723	151	108	1.40	100.0	(34)(5)
34	Lake Mead	Colorado R.	Boulder City, Nev.	167,800	13.25	1044	189	77.3	2.44	99.4	(39)(2)(4)(6)(5)(23)
35	San Carlos	Gila River	Globe, Ariz.	12,900	18.19	601	96.0	26.7	3.59	98.0	(31)(2)(4)(23)(6)
36	Conchas	So. Canadian R.	Tucumcari, N. M.	7,350	10.3	512	104	27.2	3.82	97.3[95.8][a]	(32)(36)(23)
37	Fort Peck	Missouri River	Fort Peck, Mont.	57,725	12	120	337	72.5	4.65	100.0	(38)(35)(12)
37a	Elephant Butte	Rio Grande	Hot Springs, N. M.	25,923	32.33	854	93.2	45.4	2.05	98.6	(30)(23)(2)(4)
			Desilting Basins								
[redacted]	All-American Canal	Colorado R.	Yuma, Ariz.	184,600	10.0	406	0.061	55.5	0.3011	91.7	(4)(6)
39	Hadley Creek, New	Hadley Creek	Kinderhook, Ill.	77	5.01	2490	36.7	432	0.0850	98.8	(11)
			Semi-dry Reservoirs								
40	John Martin	Arkansas R.	La Junta, Colo.	18,933	5.4	450	36.3	21.3	1.71	62.2	(4)
41	Senecaville (1936-1939)	Wills Creek	Senecaville, Ohio	121	3.2	1026	731	768	0.953	48.4	(37)(10)(6)(5)(11)

[a]Figures in brackets refer to trap efficiency with sluicing or venting operations in effect.

a short time, dredging tends to increase the amount of sediment trapped in a reservoir.

In Table 1, reservoirs and ponds of all sizes were used, ranging from a farm pond with 0.038 sq mile drainage area to Imperial Dam Reservoir, draining 184,600 sq miles. The chief criteria in selecting reliable records were accuracy with which trap efficiency could be determined and the availability of information on the various factors which affect trap efficiency. As Gottschalk stated,[8] the laws of sediment deposition are the same for all types of reservoirs, including stock ponds. Likewise, the trap efficiency of reservoirs is affected by the same factors, regardless of the size of the reservoir.

It has long been recognized[12] that capacity-inflow (C/I) ratio would be a more accurate index of trap efficiency than the capacity-watershed (C/W) ratio which has heretofore been so widely used. The C/W ratio must be used within definite hydrologic regions, and not as an index of comparison over the country as a whole, for, as is obvious, with the same C/W ratio the trap efficiency will increase as the runoff per unit area decreases.

Table 1 of Brune's paper is reproduced herein and the records have been plotted as shown in Fig. 3. Brune found that correlation for normal ponded reservoirs (conventional reservoirs as distinguished from desilting basins and dry reservoirs) operated without any special efforts at sluicing sediment, is much better using the C/I ratio than using the C/W ratio.

Records 32-37a in Table 1, and in Fig. 3, represent reservoirs such as Fort Peck Reservoir and Lake Mead which have C/I ratios greater than 1.0. In other words, these reservoirs have a capacity larger than the annual runoff from their watersheds. When the effects of evaporation and seepage are also considered, it is obvious that water will rarely be discharged over the spillways of such reservoirs, and that trap efficiency must therefore be close to 100%.

The C/I ratio thus provides a means of differentiating between the "holdover storage reservoirs" and "seasonal storage reservoirs" described by Thomas Maddock, Jr.[13] Reservoirs with a C/I ratio of 1.0 or less may be classed as seasonal storage reservoirs, and those with a ratio greater than 1.0 as holdover storage reservoirs.

Reservoirs may have very different C/W ratios and annual inflows, yet have the same C/I ratio and the same trap efficiency, other factors being equal. For instance, a reservoir in the humid part of the country may have a C/W ratio of 100, an annual inflow of 1,000 acre ft per sq-mile, and a C/I ratio of 0.1. Another reservoir in an arid region may have a C/W ratio of 10, an annual inflow of 100 acre ft per sq mile, and the same C/I ratio of 0.1. Both reservoirs, assuming they are of the normal ponded type, will have roughly the same trap efficiency (around 87%, from Fig. 3).

Theoretically, a reservoir can have a trap efficiency of zero only when its capacity is reduced to zero or the runoff approaches an infinite quantity. Also, a trap efficiency of 100% is theoretically possible only when the reservoir can contain the inflow long enough to allow all sediment to settle out. An inspection of Table 1 and Fig. 3 shows that there are several instances on record of trap efficiencies of zero and 100%.

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13 "Reservoir Problems with Respect to Sedimentation," by Thomas Maddock, Jr., pp. 9-14, Proc., Federal Inter-Agency Sedimentation Conference, Washington, D. C., Jan. 1948.

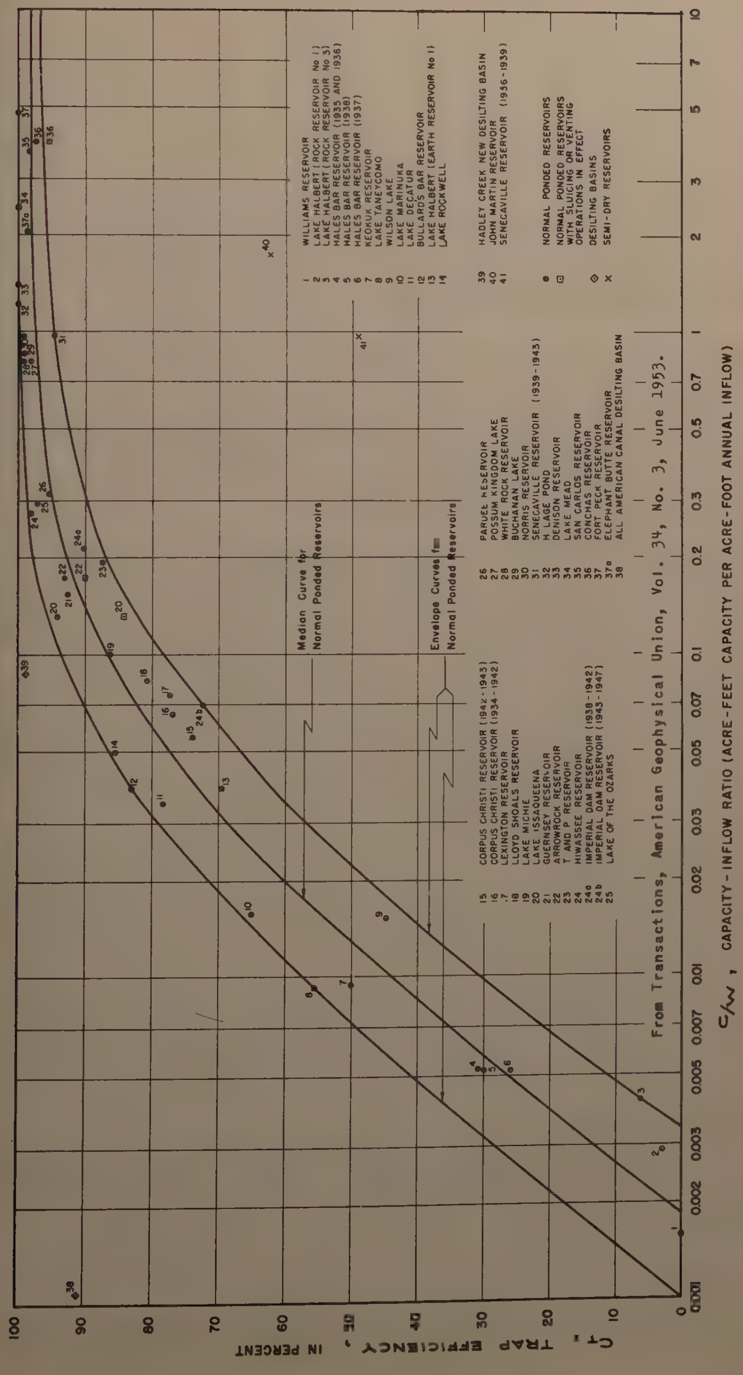

FIG. 3.—TRAP EFFICIENCY AS RELATED TO CAPACITY-INFLOW RATIO TYPE OF RESERVOIR, AND METHOD OF OPERATION

Williams Reservoir, a power reservoir near Williams, Inc., had a trap efficiency of less than zero during the period of measurement between 1930 and 1939, thus revealing that scouring actually had occurred. Three instances of trap efficiency equal to 100% were measured at H. Lage Pond, Fort Peck Reservoir, and Denison Reservoir. Details on these records may be found in Brune's paper.

Table 1 and Fig. 3 show the effects of sluicing and venting. At Arrowrock Reservoir, Idaho, sluicing is estimated to have reduced the trap efficiency from 93.0% to 90.3%, and to have increased the sediment outflow from the reservoir by 39%. Likewise, at Conchas Dam, N. M., sluicing operations decreased the estimated trap efficiency from 97.3% to 95.8%, and increased the sediment outflow from the reservoir by 56%. Those who are interested in the details of these and other sluicing and venting operations are referred to the original paper.

As pointed out by Brune, it appears that sluicing and venting operations have much better chances of accomplishing their purpose when they are timed to intercept gravity underflows as they reach the dam. If gravity underflows are not present, such operations should at least be timed to meet the higher sediment concentrations brought in by flood flows. It is likely that accurate timing to intercept gravity underflows can treble or quadruple the amount of sediment lost from the reservoir.

All of the foregoing pertains to normal ponded reservoirs. A word is now in order about desilting basins and dry reservoirs as studied by Brune. The trap efficiency of desilting basins is governed largely by their shape. Hadley Creek New Desilting Basin, Ill., (Table 1 and Fig. 3) covers 280 acres but has an average effective depth of less than 4 ft. A normal ponded reservoir with the same C/I ratio would have a trap efficiency of about 85%. This basin, however, traps 98.8% of the sediment reaching it, largely because of its shallowness and the small interval of time required for the sediment to reach the bottom. Other factors, such as flocculation, the coarse nature of the sediment, and the location of the basin at a point where the stream gradient decreases abruptly, have their effect on increasing the trap efficiency.

An even more efficient type of desilting basin is exemplified by the All-American Canal Desilting Basins, Ariz. The method of operation, in addition to the factors previously mentioned, causes these basins to have a high trap efficiency. A normal ponded reservoir with a C/I ratio as low as these basins could be expected to have a very low trap efficiency, yet the basins trap 91.7% of the sediment reaching them.

Thus, according to Brune, desilting basins may have trap efficiencies above 90% in a much lower range of C/I ratios than for normal ponded reservoirs.

Although no information is available on the trap efficiency of truly dry reservoirs, two reservoirs which have been operated in a semi-dry condition provide indicative data.

The Senecaville Reservoir, Ohio, a flood-control reservoir built by the Corps of Engineers, and the John Martin Reservoir, Colo., are examples of reservoirs being operated in a semi-dry condition. The studies on the Senecaville Reservoir revealed the trap efficiency to be 48.4% when operated with the gates wide open and 94.3% with the gates closed. The trap efficiency of the John Martin Reservoir was 62.2%, an unusually low figure for a reservoir with such a high C/I ratio (1.71).

Thus it is apparent that even holdover reservoirs like John Martin, if operated so as to allow large flows of water to pass through the dam, may have

trap efficiencies in the 60% range rather than about 90% as would be expected with normal operation. Truly dry reservoirs, such as the Miami Conservancy District reservoirs, undoubtedly have much lower trap efficiencies, probably in the 10% to 40% range, depending chiefly upon the C/I ratio.

The more important conclusions reached in the study by Brune have been summarized in the foregoing portions of the paper. However, those who are interested in more details are referred to the original paper. There are numerous references listed at the end of his paper.

UPSTREAM FLOODWATER RETARDING STRUCTURES

As pointed out previously, the only known published information available on trap efficiency is that by Brune, which has been summarized. Since much of this information applied to storage reservoirs, it is not known whether it applies also to the type of floodwater retarding structures (particularly with dry pools) being constructed by the Soil Conservation Service in the watershed protection program. For this and other reasons, the trap efficiency studies to obtain additional information for design were initiated by the Soil Conservation Service under authorization of Public Law 566 in cooperation with the U. S. Geological Survey. The studies are being executed by the Geological Survey with funds transferred by the Soil Conservation Service.

The effectiveness of floodwater retarding structures in trapping sediment from storm runoff has been observed only in a general way. The objective of the trap efficiency studies is to provide planning data for the design of such structures in the range of watershed conditions exhibited in the United States, to delineate the principal factors that influence trap efficiency, and to study the sediment yield and characteristics of small watersheds. The selection of individual projects has been made jointly by representatives of the Geological Survey and the Soil Conservation Service and has been guided by the relative need for design data, by the availability of supplementary hydrologic data and by the diversity of watershed characteristics.

Before proceeding further with the discussion of the cooperative trap-efficiency studies it might be well to discuss briefly the nature and general design of an upstream floodwater retarding structure. Fig. 4 is a section of a typical structure. The structures are rolled earth embankments with earthen vegetated spillways. They are designed without conservation pools. Their design does provide storage space for sediment, the needed space being determined by a sediment source survey of the watershed of each structure. In the design for this storage, sediment rates based on present conditions of the watershed for ten years and on treated conditions for 40 yr are used where the structure is designed to operate at full effectiveness for a 50-yr period. Depending on the character of incoming sediment, from 10% to 30% of the sediment storage is allocated in the flood pool, the remainder being provided below the inlet of the principal spillway in the sediment pool.

The principal spillway of a floodwater retarding structure usually consists of a reinforced concrete drop inlet with a heavily reinforced concrete pressure pipe, having a minimum diameter of 18 in., laid on a concrete cradle through the embankment.

The principal spillway automatically releases water from temporary storage in the detention pool at a controlled release rate. Its discharge capacity is determined from consideration of the various physical and economic factors

associated with the planning of a single dam or a system of retarding dams. Variations in discharge capacity influence detention storage time, and correlatively the trap efficiency.

A sluice gate may be constructed in the inlet of the principal spillway to permit the release of water stored below the crest pending the accumulation of sediment. This is essential in some areas to satisfy downstream water rights.

The capacity of the detention pool is such, that flow through the emergency spillway occurs not more often than once in 25 yr for any structure. Many such structures are designed with sufficient detention storage to accommodate a flood of 100-yr or more estimated frequency before the emergency spillway is brought into operation.

The sediment and flood pool volumes vary with the size of drainage area, runoff characteristics, sediment yield and other factors. In general, however, the sediment pool storage is less than 25% of the total available storage behind the structure. In the Southwest area the average volume of sediment space is 209 acre ft as compared to 1,069 acre ft of total storage, an average for 146 structures. The character of sediment, character of native water, and detention storage time appear to be the most significant parameters influencing trap efficiency.

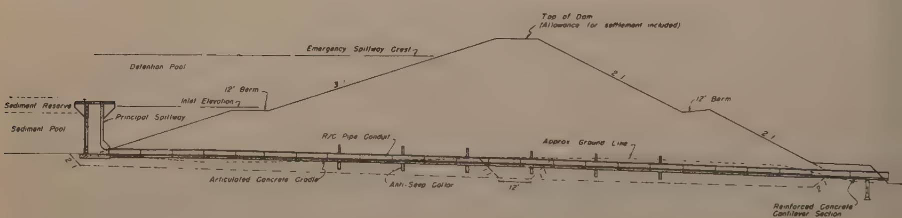

FIG. 4.—TYPICAL SECTION OF A FLOODWATER RETARDING STRUCTURE

So much for the general design of the floodwater retarding structures as it might affect trap efficiency.

The operating procedures are adjusted to the particular characteristics of each structure. Twelve structures in eleven states were selected as representative for trap-efficiency studies.[14] Information on the operation of the structures as of June 30, 1956 covers only the first ten structures. The last two structures were added after that date.

The station name, location, date established, area of watershed, capacity of the detention pool (from sediment pool elevation to the earth spillway elevation), and the detention time of each project are shown in the report in the order of their installation. The procedures at each project location are designed to provide: (1) qualitative data on the character of water and sediment flowing into the reservoir; (2) quantitative data on the amount and properties of sediment retained in the pool area; and (3) quantitative data on the amount, rate, and properties of sediment discharged from the structure. Occasional samples of inflowing storm runoff, in the order of about 10 per yr, are collect-

14 "Progress Report, Investigation of Trap Efficiency of Floodwater Retarding Structures, July 1955 to June 1956," U. S. Geological Survey to Soil Conservation Service, Feb. 11, 1957.

ed to describe the concentration and size distribution of inflowing sediment and pertinent chemical properties of the inflowing water. The Soil Conservation Service has in each instance the responsibility of making periodic pool sedimentation surveys to determine the amount, distribution and characteristics of the deposits. The frequency of pool surveys is determined by the rate of accumulation and the occurrence of unusual flood events. The amount of sediment discharged from the structure is measured by sampling at a frequency sufficient to delineate a continuous record of sediment concentration with time. Sampling frequency ranges from hourly intervals during critical periods of storm discharge to weekly intervals during periods of base flow discharge. The size distribution and flocculation characteristics of the sediment, and pertinent chemical properties of the water, are determined about ten times per year.

Supplemental data relating to precipitation, runoff, inflow-outflow relationships, physical and cultural characteristics of the watershed and physical characteristics of the structure are available in each project watershed from concurrent investigations by the Soil Conservation Service and Geological Survey.

Operation of five of the projects was begun in the interval between January and June 1956. Study of the two reservoirs in Nebraska is unique in that structure No. 2 is within the watershed of structure No. 1. At the present time only brief information covering the results of the studies is available. The establishment and testing of relationships used in computing water and sediment discharge customarily cause more lag between observation and computation of data for the initial period than for subsequent periods of operation.

Since results of measurements as of June 30, 1956, the latest available information on some of the structures, are meager due to the shortness of the period and the few runoff events since initiation of the studies, they are not being included.

However, the numerous storm events that occurred during March, April and May 1957 in Texas, Oklahoma and Arkansas provided considerable information and data relative to sediment inflow and outflow for the floodwater retarding structures on which trap-efficiency studies are being made.[15] These data are being analyzed and tabulated by the Geological Survey.

Although sufficient time has not elapsed and the storm events above the structures have not been numerous enough to give suitable data on which to design, it appears that a good start has been made to obtain reliable and accurate data needed for evaluation of trap efficiency for structures of this type. Until such time as the data does become available the Soil Conservation Service, among others, will of necessity have to adapt the presently available information for other types of reservoirs to the type of structure being considered. In this respect, indications are that, for the floodwater retarding structures of the type being designed by the Soil Conservation Service and having sediment pools such as those constructed in the Southwest area, 85% to 95% of the incoming sediment is trapped. This appears to be a reasonably conservative estimate, but will be revised as more accurate data become available.

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15 "Sedimentation Studies by the Soil Conservation Service in the Western Gulf States," by G. W. Renfro and C. M. Moore, Paper presented at National ASCE Convention, Hydraulics Division, Jackson, Miss., Feb. 20, 1957, and published in the Journal of the Hydraulics Division, ASCE, Oct. 1958, Paper No. 1806.

RESERVOIRS, DEBRIS BASINS AND DEBRIS DAMS

Before reviewing and summarizing the work being done by the Los Angeles County Flood Control District in the control of debris-laden streams, it might be helpful to know something of the background of the debris basin program in the Los Angeles area. The following information is taken from a letter report prepared in December 1952 by a member of the District's staff on the subject of "Control of Debris Laden Streams."

Southern California has experienced several severe floods in the past which have produced heavy debris flows from the mountains. One notable example is the flood resulting from the storm of December 31, 1933 - January 1, 1934.

A brush fire in November 1933 burned over an area of 4,500 acres in the mountains north of La Crescenta and Montrose about 15 miles north of Los Angeles. The mountain drainage in this area has a very rugged topography rising some 2,500 ft in a horizontal distance of 1-3/4 miles. The average slope of the debris cones at the foot of the mountains is between 8% and 10%. Individual watersheds in these mountains range from 0.2 to 1.6 sq miles in area.

During the storm mentioned, more than 12 in. of rain fell. This amount of rain falling on steep, recently burned-over areas produced great quantities of debris. It is estimated that as much as 87,000 cu yd of debris per sq mile was produced from the 4,500 acres during this storm. Although velocities of mud flows were sufficient to move many large boulders, 10 tons or more in weight, the bulk of the debris produced was probably less than 1/4 in. in size.

While a heavy rainfall on a recently denuded surface will produce great quantities of debris, a major storm can produce similar quantities from an unburned saturated area. For example, the storm of March 2, 1938 fell on fairly well saturated ground and produced about 72,000 cu yd of debris per sq mile from the same area. One area of 0.27 sq miles produced debris at a rate of 120,000 cu yd per sq mile.

At the time of the flood of January 1, 1934 a gravel pit existed near the mouth of Haines Canyon. This pit had been provided with a temporary spillway to investigate its action as a debris basin, collecting the debris and allowing desilted water to pass on down the channel. The debris damage downstream from this pit was not as severe as in the areas not similarly protected.

With this example before it, the Los Angeles County Flood Control District in 1934 embarked on a program of debris basin construction. Later the U. S. Army Engineers undertook the construction of additional basins in the Montrose-La Crescenta area.

Essentially, this system consists of basins located at the mouths of potentially hazardous canyons and of concrete-lined lateral channels leading from the basins to a major collector channel. The lateral channels being located on debris cones, are naturally on steep grades and consequently carry the flow at high (super-critical) velocities. Careful attention must therefore be given to super-elevation at curves and to oscillation of the water prism thereby created. Rectangular reinforced concrete channels have been found to be the most satisfactory for these conditions of flow.

As mentioned, the New Year's storm of January 1, 1934, produced about 87,000 cu yd of debris per sq mile of watershed. This led to the adoption of a design capacity value of 100,000 cu yd per sq mile for debris basins in the Montrose-La Crescenta area. This has been the accepted design value when a permanent installation is considered.

However, since debris production is dependent upon a number of variables such as rainfall intensity, slope of watershed, vegetation and soil conditions, it has been the experience of the District that the rate of debris production per square mile for areas other than the Montrose-La Crescenta area can range between 40,000 and 10,000 cu yds per sq mile. In view of this wide variation, some means of estimating previous amounts of debris flows from a given watershed provides the most satisfactory means of establishing production rates. At present there is no completely reliable means of using known debris production rates for a given watershed in determining the production rates for a different watershed, except by a general comparison of topographic features and rainfall characteristics.

The basins themselves have been of two types, (1) excavated, and (2) nonexcavated.

The excavated basin is obtained by digging a pit of the required capacity and using the material removed to form a dam of suitable curvature around the downstream part of the basin. In this dam an adequate concrete spillway structure 6 ft to 10 ft in height is constructed, generally in line with and having its foundation at the existing streambed. Several uncontrolled discharge ports are generally incorporated in the spillway to allow for some release during storms, and partial post-storm drainage. It is essential that an intake structure be provided from the normal streambed to the bottom of the excavated pit. Otherwise, channel erosion will occur at the inlet to the basin and progress upstream. A line connecting the lips of the inlet and spillway structures is generally at a slope that is about one-half to two thirds of the natural slope of the stream channel. These basins are particularly appropriate where complete desilting, at least up to the ports, is desirable.

The unexcavated debris basin is similar to a small storage reservoir, with a few variations. The dam is generally a compacted earthfill with heavy rock armor or reinforced concrete on its upstream slope. For purposes of estimating the storage capacities, the spillway lip is at an elevation that will create the desired debris storage capacity. The debris slope is assumed to be about one-half to two-thirds of the natural channel slope upstream from the spillway lip. No inlet structure is required for an unexcavated basin. A provision is generally made to allow for some release during storms. This may be done by means of ports through the spillway or some other outlet structure. These basins lend themselves to partial desilting, that is, to the retention of debris of such size only as would be detrimental to channel improvements downstream from the basins if allowed to be discharged therefrom. This may be accomplished by designing the width and slope of the basin so as to cause the desired segregation of debris to take place automatically.

It has been the policy of the Los Angeles Flood Control District to limit debris accumulation to 25% of the original basin capacity before debris removal is considered necessary. There is no fixed rule for removal, however, for a rather flexible basis has been used in the operation and maintenance of the debris control system. The allowable debris accumulation is considered in the light of the probable debris production and the condition of the watershed relative to vegetation and geological characteristics.

Debris disposal presents a major problem in rapidly growing southern California. Generally speaking, the accumulated debris has no agricultural value in spite of the very high percentage of fine material. After a number of years of operation of a debris control system with the consequent debris removal, the nearby disposal areas are exhausted and more remote sites must

be found. This makes mechanical debris removal a costly procedure. It would be desirable to segregate the bulk of the fine debris and cause it to be discharged with the water, as previously discussed. This type of debris removal eliminates the excavation cost of the accumulated debris and minimizes interference with the normal replenishment of sand for the beaches of southern California.

In general, floating debris does not present a great problem for the debris basins. The watersheds are usually quite small and the volume of this type of debris is not large. However, until the water surface in the basin rises above the ports, there is always the possibility of clogging. Patrolmen have been able to keep the ports clear more or less satisfactorily during storms. The District has been experimenting with inclined trash racks made from railroad rails and spaced roughly one-half of the minimum dimension of the ports. The results have been satisfactory so far, though incomplete. When the water overflows the spillway lip the floating debris is allowed to pass on down the channel. At the end of the runoff season when the basin has been allowed to drain, the remainder of the floating debris accumulates on the sides and bottom. It is then either collected and burned or removed from the basin by trucking.

At times it is necessary to construct temporary debris basins and channels before sufficient funds are available to build permanent structures. Under these conditions the design capacity, 100,000 cu yd per sq mile, is sometimes reduced to 50,000 cu yd per sq mile, or less. Such a basin might have asphalt-lined or wooden inlet and spillway structures, as well as an asphalt-lined discharge channel. The use of asphalt to replace concrete requires caution, particularly where impact due to coarse debris and boulders is concerned. Asphalt will not stand up under such punishment. It will, however, resist abrasion due to ordinary drag of a bed load of debris, the maximum size of which is of the order of three inches.

Necessarily, any flood protection must be based on a definite plan of control. Portions of an area might require storage reservoirs for debris control and flood flow regulation while smaller watersheds may need only a system of debris basins for satisfactory protection.

Since December 1952 the most important modification of the early debris basin concept and design has been the elimination of dead storage and of the open port outlets which were provided in addition to the spillway in many of the debris basins, and the substitution of an outlet tower. This is located at the low point in the bottom of the debris basin offset from the line of flow between the inlet and the spillway and serves as the entrance to a conduit which passes under the dam and discharges in the channel downstream. This outlet design accomplishes two things: (1) It permits storm flows which are relatively free of debris to pass on through the basin with a minimum of ponding, thus leaving basin capacity for debris storage, and (2) it serves as a trash rack. These towers have been constructed in circular form of 5-ft inside diameter of reinforced concrete or reinforced gunite. They have a 6-in. wall thickness with a series of round perforations four inches in diameter on the outside and five inches in diameter on the inside face of the tower. The tower is of sufficient height to project at least one foot above theoretical debris slope, equal to 75% of the original streambed slope. The outlet pipe normally is a 36-in. reinforced concrete pipe with a slope of not less than 5% and a capacity of at least 150 cfs for the maximum expected head of water in the basin. By means of a controlling orifice at the base of the tower, flow under pressure in the outlet is prevented. Other modifications in design have no specific bearing on the subject of trap efficiency and need not be discussed here.

In presenting information relative to the reservoirs, debris basins and debris dams of the Los Angeles County Flood Control District, it appeared that perhaps the best way would be to follow the format of Table 1 of Brune's paper.[4] This procedure was followed insofar as the data permitted. Tables entitled "Reservoir Debris Production Data," Table 2, and "Debris Dams and Debris Basins—Debris Production Data," Table 3, are included herein.

From the data available it was not possible to include three columns which were included in Brune's paper. These were (1) average annual inflow, (2) C/I^b, and (3) trap efficiency. This omission may be explained by stating that relative to debris basins and the majority of the debris dams, continuous inflow and outflow records are not available. In fact, very few records of inflow and outflow are available. Due to the lack of inflow records, the values for C/I^b and trap efficiency could not be computed. However, it is believed that in nearly all cases the trap efficiency, at least of course-grained debris, approaches 100%. With the exception of five of the basins, namely, Aliso Creek, Scholl, Snover, Ward Canyon, and Verdugo, the material is trapped, as will be later explained. In the abovementioned basins, the silt has been allowed to accumulate to the level of outlets. Much of the fine material and that which is in suspension has been allowed to go on through. This has been somewhat in the nature of an experiment, but so far appears to be a successful plan of operation for these basins, at least during seasons where the watersheds are not burned off and no subsequent floods occur. In the case of Scholl and Ward Canyon, flashboards are placed in the outlet notches to regulate to some extent the level of debris in the basin. At the other basins, except in the case of sizable floods which fill the basins with sediment, all material and most of the inflow is trapped during a given storm. If, subsequent to a storm, it is found that less than 75% storage capacity remains, the material is removed by shovel and trucks to a nearby disposal area.

The omission of the three-above-mentioned columns in the tabulation for flood control dams is due to the fact that although the District does have inflow and outflow records, sediment generally goes through the outlets only during periods of controlled sluicing. Under certain other conditions such as during flood releases, small quantities of extremely fine silt and sand may pass through the valves. Normally flood releases are made only when an appreciable head exists on the valve. Other releases are made subsequent to storm periods and therefore after most of the suspended material has settled out. As stated above, the District believes that trap efficiency for the reservoirs generally approaches 100%, even during periods when both inflow and outflow take place. As the reservoirs serve the dual purpose of flood control and water conservation, water is stored to limits compatible with flood operation.

In some of the reservoirs where conditions permit, capacity is regained by the hauling of some of the silt accumulation by permittees. In general, this does not constitute as large a storage reclamation as is gained by sluicing, as may be noted from the table. Sluicing, of course, is dependent upon sufficient residual flows. This is often a problem, as the reservoirs may not be drawn down due to conservation commitments until late summer or fall when inflows are low.

A study has recently been completed by the District's Foundation and Testing Division, relative to the quality of debris excavated from the debris basins and removed to disposal areas; the latter have been provided and set aside by the District exclusively for disposal of debris. Inasmuch as the size of material trapped by the debris basins is of interest and has a bearing on the subject of trap efficiency, a study was made of grading analyses for material from

TABLE 2.—RESERVOIR DEBRIS PRODUCTION FOR THE LOS ANGELES COUNTY FLOOD CONTROL DISTRICT

Dams and Reservoirs	Location[a]	Original Spillway Capacity in Acre-Feet	Drainage Area, in Square Miles	Total Number of Debris Seasons	Quantity Sluiced in Acre-Feet	Quantity Excavated in Acre-Feet	Volume of Debris in Storage in Acre-Feet	Total Debris Inflow, in Acre-Feet	Average Annual Debris Production per Square Mile in Acre-Feet	C/W, in Acre-Feet or Square Mile[c]	Average Annual Inflow, in Acre-Feet or Square Mile
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1. Big Dalton	Big Dalton Canyon - 4 mi. N.E. of Glendora	1,053	4.49	15.0	0	0	101	101	1.50	234.5	709
2. Big Tujunga	Big Tujunga Canyon - 10 mi. N.E. of Sunland	6,240	82.3	23.0	1210	0	2,141	3,351	1.77	75.8	14,404
3. Cogswell	West Fork San Gabriel Canyon - - 22 mi. N.E. of Azusa	12,298	39.2	11.92	564	0	1,664	2,228	4.77	313.7	17,653
4. Devils Gate	On Arroyo Seco - N.W. Pasadena	4,601	(31.9) 15.1[d,e]	35.92	1040	293	1,892	3,225	3.50	304.7	3,962
5. Eaton Wash	Eaton Wash - N.E. of Pasadena	956	9.48	15.25	41	243	253	537	3.71	100.8	1,654
6. Live Oak	Live Oak Canyon - 2.5 mi. N.E. of La Verne	250	2.28	30.08	0	0	29	29	0.42	109.6	157
7. Pacoima	Pacoima Canyon - 4 mi. N.E. of San Fernando	6,060	28.2	28.0	170	0	1,273	1,443	1.83	214.9	6,157
8. Puddingstone	Puddingstone Creek - 1 mi. S. of San Dimas	17,398	(32.1) 11.0[e]	13.25	0	0	208	208	1.43	1581.6	2,258
9. Puddingstone Div.	San Dimas Creek - 2 mi. N.E. of San Dimas	148	(18.84) 2.64	24.92	0	122	9	131	1.99	56.1	1,724
10. San Dimas	San Dimas Canyon - 3 mi. N.E. of San Dimas	1,496	16.2	33.0	34	0	471	505	0.94	92.3	2,606
11. San Gabriel	San Gabriel Canyon - 7 1/4 mi. N. of Azusa	53,344	(202.7) 163.5[e]	16.67	2002	0	9,331	11,333	4.16	326.3	103,164
12. Santa Anita	Santa Anita Canyon - 2 1/2 mi. N. of Arcadia	1,376	10.8	29.33	617	0	763	1,380	4.36	127.4	4,266
13. Sawpit	Sawpit Canyon - 2 mi. N. of Monrovia	476	3.34	27.42	57	0	171	228	2.49	142.5	[illegible]
14. Thompson Creek	Thompson Creek - 3 mi. No of Claremont	645	3.51	29.69	0	29	79	108	1.04	183.8	167
	TOTALS	106,341	392.04		5735	687	18,385	24,807			

[a]All in Los Angeles County, California. [b]Excluding any material passed through except that sluiced. [c]Original spillway capacity divided by uncontrolled drainage area. [d]Due to construction of upstream debris-control structures, the uncontrolled drainage area has varied from 31.9 sq miles originally to 15.1 sq miles as of the last survey. [e]Figure in parentheses indicates total watershed area controlled by this and other flood-regulating structures.

TABLE 3.—DEBRIS PRODUCTION DATA FOR DEBRIS DAMS AND DEBRIS BASINS OF THE LOS ANGELES COUNTY FLOOD CONTROL DISTRICT

Debris Dams	Location[a]	Total Drainage Area, in Square Miles	Uncontrolled Drainage Area, in Square Miles	Period of Record, in Years	Total Volume of Debris Excavated or Sluiced in Cubic Yards	Debris in Storage, in Cubic Yards	Total Debris Inflow, in Cubic Yards[b]	Average Annual Debris Production per Square Mile of Drainage Area, in Cubic Yards[c]	Maximum Debris Storage Capacity, in Cubic Yards	C/W, in Cubic Yards per Square Mile[d]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1. Bailey	Sierra Madre		0.60	12	36,894	2,948	39,842	5,534	165,668	276,113
2. Lower Big Dalton	Glendora		0.34	3	0	0			27,519	80,938
3. Rubio	Altadena	1.71	1.26	14	500	9,669	10,169	576	144,324	114,543
4. Sawpit	Monrovia		2.84	3	0	10,600	10,600	1,244	746.020	262,683
5. Sierra Madre	Sierra Madre		2.39	30	155,487	10,414	165,901	2,314	151,820	63,523
6. Sunset	Burbank		0.44	28	12,329	5,678	18,007[f]	1,462	17,498	39,768
7. Verdugo	Verdugo City	15.50	9.43	22	371,812	28,444	400,256[f]	1,929	151,684	16,085

Debris Basins										
1. Aliso Crk.—Wilbur Ave.	Northridge		8.63	15	213,521	12,828	226,349[f]	1,749	40,463	4,689
2. Altadena G. C.	Altadena	2.36	0.65	12	16,256	2,070	18,326[f]	2,349	14,947	22,995
3. Auburn	Sierra Madre		0.19	3	[illegible]	1,131	1,131	1,984	57,074	300,389
4. Bradbury	Monrovia		0.68	3	0	6,510	6,510	3,191	53,153	78,166
5. Brand On.	Glendale		1.03	22	4,827	9,529	14,356	634	72,486	70,375
6. Carter	Sierra Madre		0.12	3	0	1,390	1,390	3,861	21,842	182,017
7. Cooks On.	Tujunga		0.58	6	20,653	2,244	22,897	6,580	47,483	81,867
8. Deer On.	Glendale		0.59	3	[illegible]	Negligible	Negligible	Negligible	44,624	75,634
9. Dunsmuir On.	La Crescenta		0.84	21	131,949	17,112	149,061	8,450	123,684	147,243
10. Eagle On.	La Crescenta		0.61	21	76,918	20,864	97,782	7,633	75,684	124,072
11. Fair Oaks	Altadena		0.21	22	55,270	1,335	56,605	12,252	28,466	135,552
12. Fern On.	Altadena		0.30	22	80,573	1,742	82,315	12,472	32,911	109,703
13. Floral, Upper	Sierra Madre		0.06	3	[illegible]	900	900	5,000	2,252	37,533
14. Floral, Lower	Sierra Madre	0.17	0.11	3	Negligible	Negligible	Negligible	Negligible	5,399	49,082
15. Gould On.	La Canada		0.47	10	0	10,532	10,532	2,241	53,848	114,570
16. Haines On.	Tujunga		1.53	22	96,284	25,952	122,236	3,631	158,595	103,657
17. Halls On.	La Crescenta		1.06	22	267,931	37,577	305,508	13,101	95,821	90,397
18. Hay On.	Alta Canyada		0.20	21	23,876	5,037	28,913	6,884	39,750	198,750
19. Lannan	Sierra Madre		0.25	3	[illegible]	8,991	8,991	11,988	56,482	225,928
20. Las Flores On.	Altadena		0.45	22	80,333	6,058	86,391	8,726	61,634	136,964
21. La Tuna On.	Sun Valley		5.34	2	16,836	11,250	28,086	2,630	70,200	13,146
22. Lincoln	Altadena		0.50	22	69,657	2,260	71,917	6,538	40,858	81,716
23. Maddock	Duarte		0.25	3	5,808	[illegible]	5,808	7,744	29,930	119,720
24. May No. 1	Sylmar		0.70	4	0	1,963	1,963	701	67,080	95,829
25. May No. 2	Sylmar		0.09	4	0	Negligible	Negligible	Negligible	6,719	74,656
26. McClure	Burbank		0.62	4	[illegible]	15,789	15,789	6,367	109,824	177,135
27. Nichols On.	Hollywood		0.94	20	54,343	7,607	61,950	3,295	32,220	34,277
28. Paradise On.	La Canada	0.94	0.47[e]	13	15,107	1,739	16,846	2,757	14,784	31,455
29. Pickens On.	La Crescenta		1.84	22	270,367	52,518	322,885	7,976	116,476	63,302
30. Rowley	Tujunga		0.58	4	0	1,948	1,948	840	37,289	64,291
31. Ruby On., Upper	Monrovia		0.21	4	0	1,836	1,836	2,186	15,031	71,576
32. Ruby On.	Monrovia	0.52	0.31	2	0	820	820	1,323	32,160	103,742
33. Scholl	Eagle Rock		0.66	12	[illegible]	3,776	3,776[f]	477	30,945	46,886
34. Schields On.	La Crescenta		0.27	20	65.262	4,950	72,212	13,002	46,936	173,837
35. Snover On.	La Crescenta		0.23	21	35,442	17,619	53,061[f]	10,986	37,941	164,961
36. Sparr	Glendale		0.84	11	7,822	0	7,822	847	15,305	18,220
37. Stough	Burbank		1.65	17	57,568	6,447	64,015	2,282	103,695	62,845
38. Turnbull On.	Whittier		0.99	5	0	2,067	2,067	418	26,657	26,926
39. Ward On., Upper	La Crescenta		0.10	1	3,894	1,160	5,054	50,540	10,785	107,850
40. Ward On.	La Crescenta	0.64	0.54	13	25,995	860	26,855[f]	3,825	10,402	19,263
41. West Ravine	Altadena		0.25	22	99,019	1,919	100,938	18,352	49,598	198,392
42. Zachau	Tujunga		0.35	1	[illegible]	Negligible	Negligible	Negligible	37,153	106,151
TOTALS					2,372,533	376,083	2,748,616		3,433,119	

[a]All in Los Angeles County, California. [b]Debris excavated plus debris in storage (does not include fines in suspension in spillway discharge. [c]Based on complete seasons. [d]C is the maximum debris storage capacity and is the amount of debris that can be stored below a plane extending upstream on a slope that is approximately 0.6 of the natural stream-bed slope upstream from the dam. W is the uncontrolled drainage area. [e]Due to construction of Gould Debris Basin and interception of flow by subdivision development, the uncontrolled drainage area has varied from 1.07 sq miles to 0.47 sq miles. [f]This figure does not indicate total amount of debris produced to date because of an unknown amount of debris carried through V-notch in spillway, open ports in spillway ogee or dam, or vertical slot through spillway.

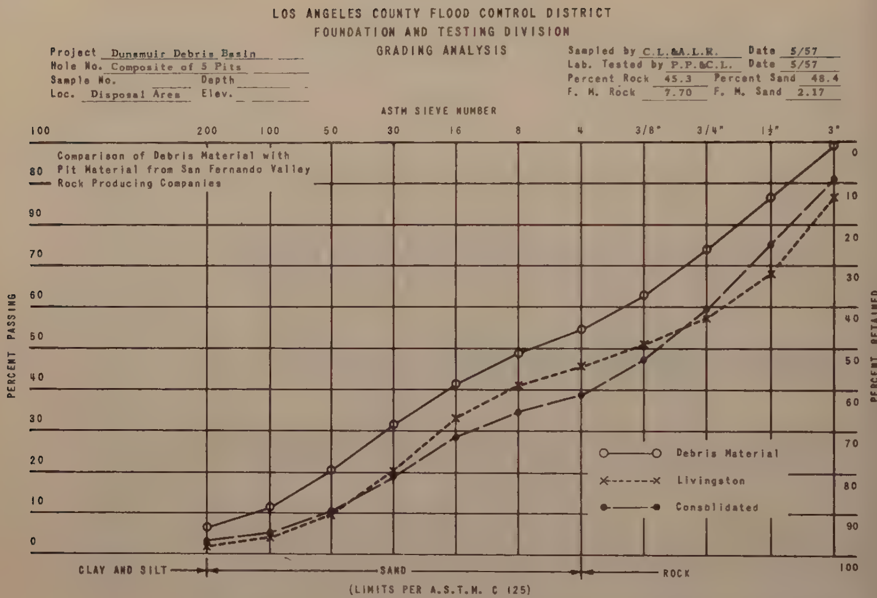
LOS ANGELES COUNTY FLOOD CONTROL DISTRICT
FOUNDATION AND TESTING DIVISION
GRADING ANALYSIS
Project Dunsmuir Debris Basin
Hole No. Composite of 5 Pits
Sample No. Depth
Loc. Disposal Area Elev.
Sampled by C.L.&A.L.R. Date 5/57
Lab. Tested by P.P.&C.L. Date 5/57
Percent Rock 45.3 Percent Sand 48.4
F. M. Rock 7.70 F. M. Sand 2.17
ASTM SIEVE NUMBER
Comparison of Debris Material with Pit Material from San Fernando Valley Rock Producing Companies
PERCENT PASSING
PERCENT RETAINED
Debris Material
Livingston
Consolidated
CLAY AND SILT
SAND
ROCK
(LIMITS PER A.S.T.M. C 125)

FIG. 5.

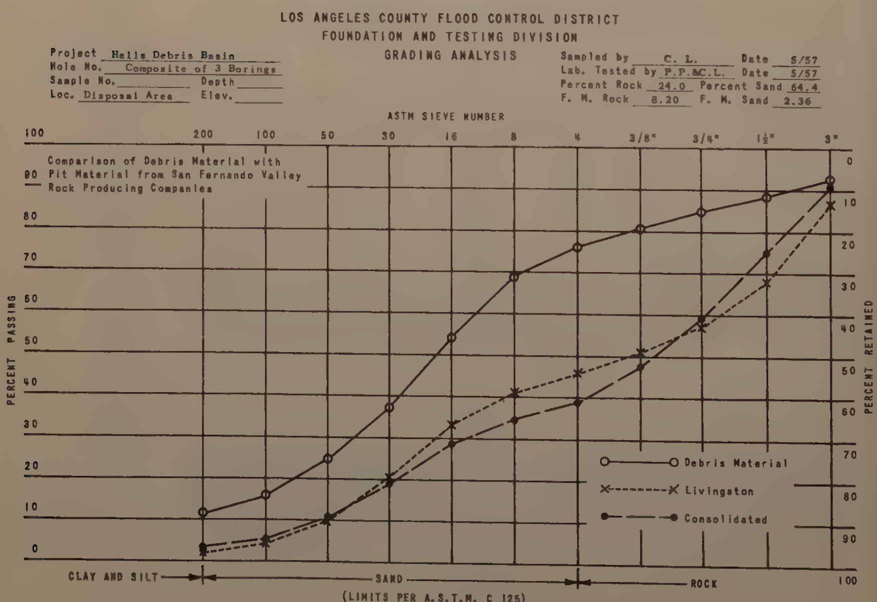
LOS ANGELES COUNTY FLOOD CONTROL DISTRICT
FOUNDATION AND TESTING DIVISION
GRADING ANALYSIS
Project Hails Debris Basin
Hole No. Composite of 3 Borings
Sample No. Depth
Loc. Disposal Area Elev.
Sampled by C. L. Date 5/57
Lab. Tested by P.P.&C.L. Date 5/57
Percent Rock 24.0 Percent Sand 64.4
F. M. Rock 8.20 F. M. Sand 2.36
ASTM SIEVE NUMBER
Comparison of Debris Material with Pit Material from San Fernando Valley Rock Producing Companies
PERCENT PASSING
PERCENT RETAINED
Debris Material
Livingston
Consolidated
CLAY AND SILT
SAND
ROCK
(LIMITS PER A.S.T.M. C 125)

FIG. 6.

basins including Cooks, Dunsmuir, Halls, Pickens, Ward, and West Ravine. The grading analyses for the various basins were compared with pit material from the San Fernando Valley rock producing companies. Figs. 5 and 6 are typical gradation curves for material from the Dunsmuir and Halls debris basins.

The rock content of the debris material found in the various disposal sites varied from 10% to 45%, with an average of 29%. This is considerably less than the average of about 58% rock found in the commercial material.

In general, the gradation of the rock was considerably finer than that of the commercial sources. Excluding the material from West Ravine, Halls, and Aliso-Wilbur debris basins, the debris rock compared favorably with that of the commercial sources relative to abrasive loss.

The debris material from the various disposal sites contained between 48% and 74% sand, with an average of 63%. This is greater than the average of 40% sand found in the commercial material. The quality of the debris sand compared favorably with the commercial material.

In order to determine more accurately the degree of trap efficiency of the Los Angeles County Flood Control District structures, it will be necessary to obtain more data directly related to the subject. These data should include continuous inflow and outflow records and measurements of sediment coming into and passing through the structures. If such data were available, the trap efficiency could be computed on a C/I ratio basis and perhaps a correlation could be made for these structures as has been done for normal ponded reservoirs.

Journal of the

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

STRATEGIC ASPECTS OF URBAN FLOOD PLAIN OCCUPANCE[a]

By Gilbert F. White[1]

SYNOPSIS

The paradox of continuing flood control and rising flood losses is explained in part by elements entering into decisions to change urban flood-plain occupance, 1936-1957. Curbing the mounting toll calls for public action to broaden the range of choice open to managers of flood-plain properties in adjusting to flood hazard.

INTRODUCTION

A striking feature of national efforts at flood control in the United States is that in the more than two decades since a national policy was launched in 1936, the mean annual toll of flood losses has continued to rise. While at least 4 billion dollars have been spent for engineering works to reduce and control floods, the economic losses from occasional flood disaster have mounted. To understand this paradox it is necessary to examine what has been happening in the human occupance of flood plains. To find out the full import of changes in flood-plain occupance it is essential to identify those factors which enter into both public and private decisions as to flood-plain use.

This paper outlines briefly the broad problem of flood-loss reduction in the United States, reviews the findings from a recent study of changes in urban occupance of flood plains, and then suggests three strategic aspects of such occupance that seem likely to affect the results of any further engineering efforts at flood control.

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. HY 2, February, 1960.

[a] Presented at the July, 1959 ASCE Hydr. Conf. in Fort Collins, Col.

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The broad problem of flood-loss reduction is that the rate at which floo losses are being eliminated by construction of engineering or land-treatmen works is of about the same magnitude as the rate at which new property is be- ing subjected to damage. Even though the data on which estimates of nationa flood loss are based seem too inaccurate to warrant any precise comparison o the two rates, it is clear, both from the aggregated damage statistics and fro the record of selected flood plains, that the heavy investments in flood protec-

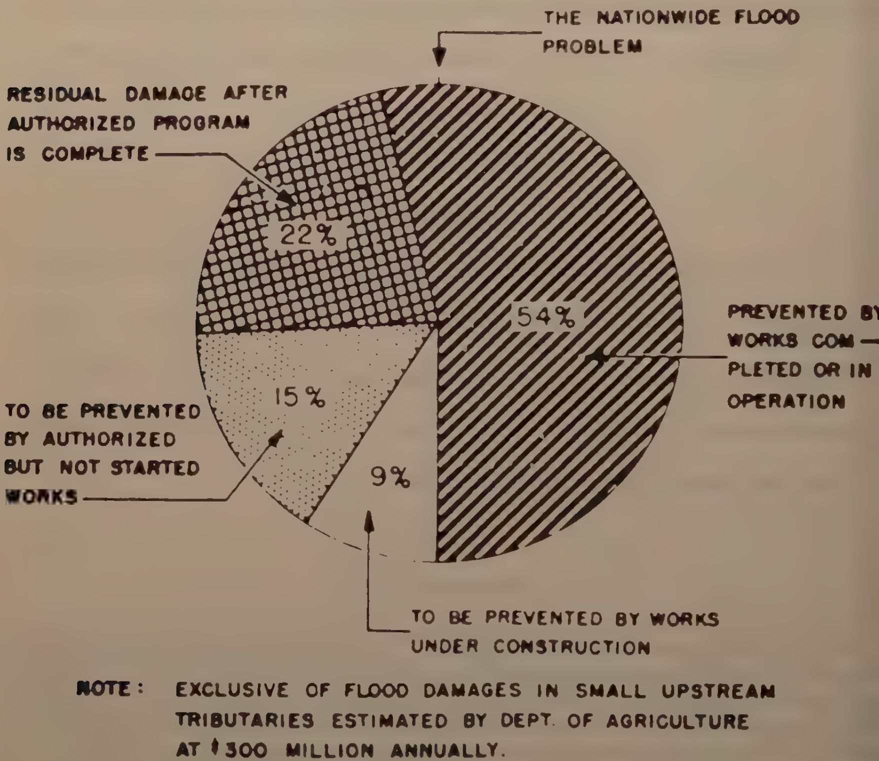

FIG. 1.—GRAPH PRESENTED BY THE CORPS OF ENGINEERS TO THE HOUSE COMMITTEE ON PUBLIC WORKS IN 1957 SHOWING AN ESTIMATE OF MEAN ANNUAL FLOOD-DAMAGE POTENTIAL AND OF DAMAGES PREVENTED. (FROM 85TH CONGRESS, 1ST SESSION, HOUSE COMMITTEE ON PUBLIC WORKS, PRINT NO. 1, 1957).

tion have effectively curbed the losses in many areas but that new damage po tention is being built up at the same time.

Studies of seventeen selected urban areas having flood problems reveal general and persistent encroachment of urban structures upon the flood plain during the period 1936-1957, even in areas in which there was net decrease i total population. They show certain distinctive patterns of encroachment, an indicate highway construction and flood-control works as two major stimulant to growth. The situations at Boulder, Colo. and Denver, Colo., illustrate som of the major findings.

In the light of that experience it is argued that at least three aspects or urban occupance not ordinarily considered in flood-control plans in the past must be taken into account in the future if the tide of rising flood losses is to be turned. First, it must be recognized that engineering works are only one of the possible human adjustments to flood hazard. Second, it must be seen that the complex of elements entering into decisions as to future occupance of flood plains includes many considerations in addition to the traditional one of cost-benefit evaluation. Third, the range of choice now permitted property managers in dealing with the flood hazard is so restricted that radical changes must be made in public policy to broaden the range of choice among possible adjustments and to assure a full appraisal of each choice.

TRENDS IN FLOOD LOSSES

There are two major sources for national estimates of flood losses. The Corps of Engineers issues, from time to time, an estimate of the total potential flood losses and of the losses that have been prevented or will be prevented by works constructed or planned by the Corps. A sample graphical statement of such an estimate for 1954 is given in Fig. 1. As of that year the Corps found a potential mean annual loss of 964 million dollars.[2] (All estimates are adjusted to 1957 price levels.) After all works then authorized had been completed there would remain a balance of approximately 210 million dollars that would not be prevented. These losses are computed separately for each of the major hydrographic areas of the country (Fig. 2).

The Weather Bureau issues the other series of flood-loss statistics, having published each year since 1903, an estimate of reported losses for each of its districts. The national totals for 1902-1955 are shown in the lower part of the bars in the graph in Fig. 3. These totals have been adjusted to 1957 price levels, and the full height of the bar shows the totals on a comparable basis.

No attempt will be made here to appraise the validity and the discrepancies among these two sets of estimates and other less comprehensive series. This has been done in a separate publication.[3] Aggregate data alone are unsatisfactory and need to be checked in more detailed fashion. It is sufficient here to point out one characteristic which the two national estimates have in common. Both suggest a pronounced upward trend in the size of annual flood losses since 1936, when the first national flood control legislation was enacted. The estimated mean annual losses in 1936, were about 212 million dollars. The Corps estimate of remaining annual losses after allowing for works completed was 444 million in 1954, and 700 million in 1959, (at 1959 prices).[4] The Weather Bureau estimates of mean annual losses for 1924-1953 were 25% lower than those for 1944-1955.

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2 "House Committee Print," U. S. Congress, House Committee on Pub. Works, 1957, No. 1, 85th Congress, 1st Sess. p. 2.

3 "Changes in Urban Occupance of Flood Plains in the United States," Gilbert F. White, Wesley C. Calef, James W. Hudson, Harold M. Mayer, John R. Sheaffer, and Donald J. Volk, Univ. of Chicago Geography Research Paper, 1958, No. 57, pp. 1-19.

4 "Hearings on Civil Works Appropriations Bill," U. S. Congress, House Appropriations Committee, 1959, Pt. I, p. 18.

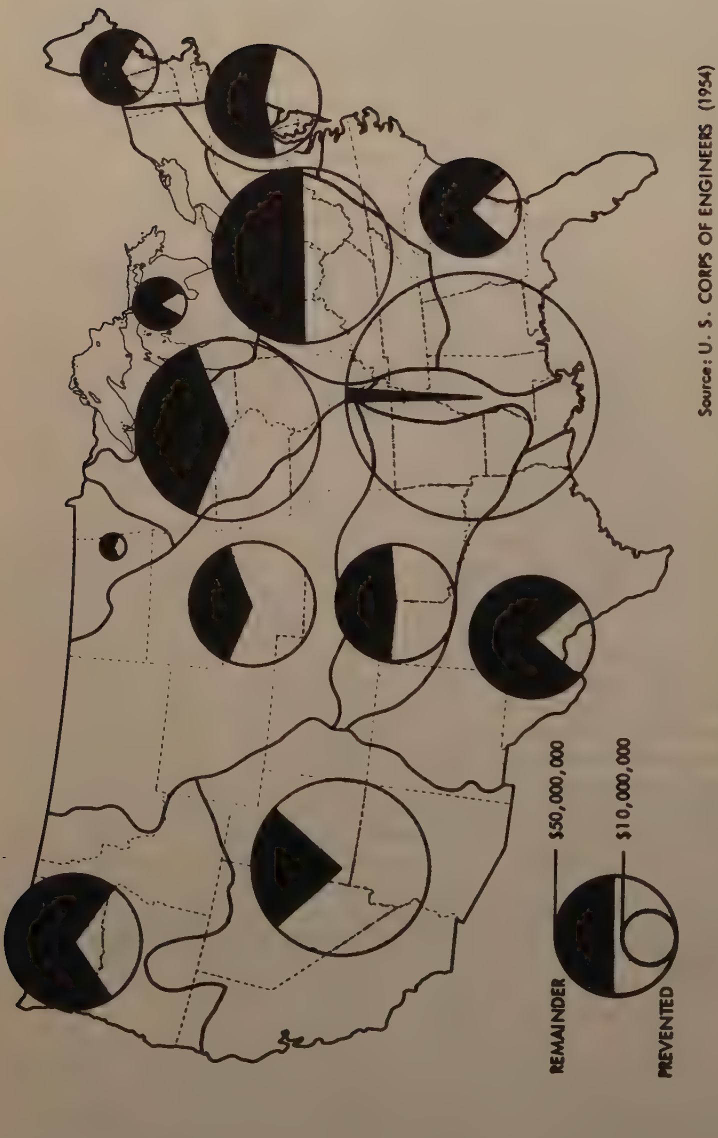

FIG. 2.—MAP SHOWING ESTIMATED MEAN ANNUAL FLOOD LOSSES AND LOSSES PREVENTED IN THE UNITED STATES (DATA FROM 85TH CONGRESS, 1ST SESSION, HOUSE COMMITTEE ON PUBLIC WORKS, PRINT NO. 1, 1957).

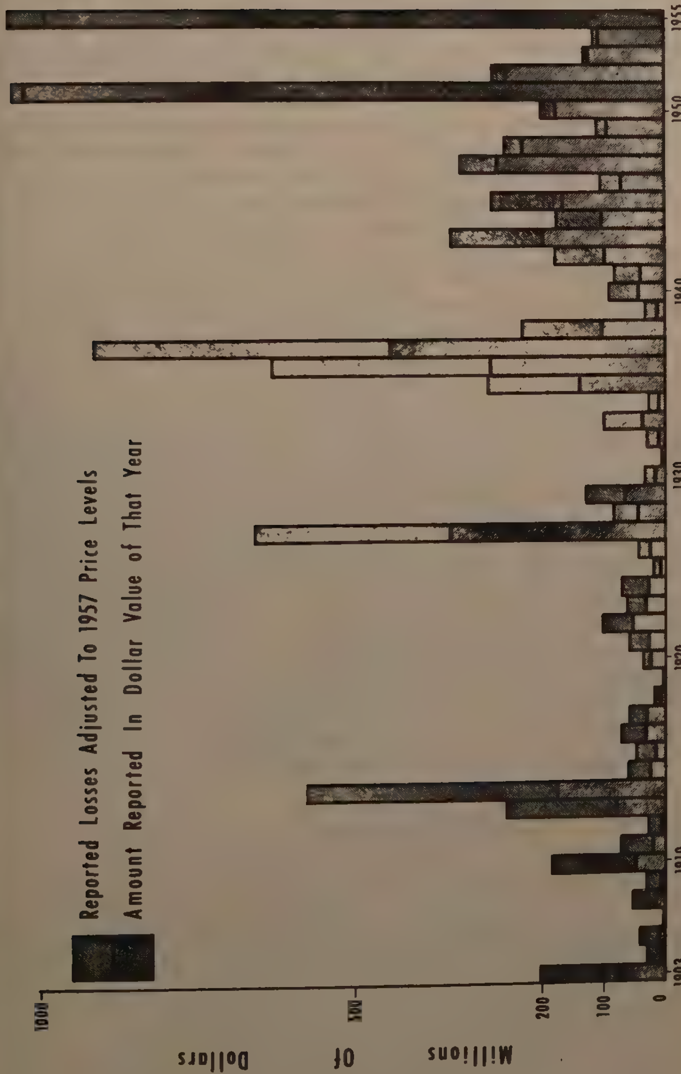

FIG. 3.—ESTIMATED ANNUAL FLOOD LOSSES IN THE UNITED STATES, 1903-1955, WITH AMOUNTS ADJUSTED TO 1957 PRICE LEVELS (DATA FROM U.S. WEATHER BUREAU).

REASONS FOR MOUNTING FLOOD LOSSES

One partial explanation for the apparently rising toll of recorded losses is that the inflation in dollar values has exaggerated the size of recent estimates. As shown in Fig. 3, even when values are adjusted to a common year there still remains a clear and sizeable increase.

A second reason advanced for the apparently mounting losses is improvement in enumeration and estimating methods. This unquestionably has played a role as Weather Bureau procedures have been made more uniform and as Corps of Engineers studies, have reached areas not previously covered. Some who have been studying this problem believe that changes in enumeration methods may account for as much as 10% to 15% of the increase.

Perhaps of greater importance is the difference in number of large and infrequent floods. William G. Hoyt, F., ASCE and Walter B. Langbein, F., ASCE have shown a 35% increase in their "flood index" from the first 25 yr. of the century to the second 25 yr.[5] There was a remarkable bunching of rare flows in the lower Missouri and Northeastern basins during the 1950's. This hydrologic record may account for as much as 25% of the increase in losses.

Probably the most important reason for the rising trend in flood losses is to be found in the continuing encroachment of human occupance upon flood plains. This takes the form of new structures, of changes in the intensity of existing structures, and of structures which so reduce the hydraulic efficiency of valley sections as to increase the hazard in affected reaches of the stream. Although there have been numerous studies of flood-control projects there has been no comprehensive investigation of the changes that have been occurring in the areas subject to flood. One recent study conducted by University of Chicago geographers dealt with urban occupance because it has a high and increasing proportion of flood losses, is compact and is more susceptible to change. It shows that there are at least 1,020 urban places with a population of more than 1,000 that have well defined flood problems. That study also gives a precise picture of the changes in seventeen urban areas selected for their diversity in valley section, flood frequency and height, size, population growth, and types of land use. A few findings from those field examinations will be summarized, and then illustrated from the Boulder and Denver situations.

GROWTH PATTERNS

The most evident and widespread trend that is to be observed in the urban flood plains is one of growth. In every place studied, including several in which the total population declined during the 21-yr. period, the number of structural units in the flood plain increased.[6] The growth rate ranged from less than 2% in a stable city like Wheeling, W. Va., to more than 600% in a rapidly expanding place such as Dallas, Tex.

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5 "Floods," by William G. Hoyt, and Walter B. Langbein, Princeton Univ. Press, 1955.

6 Changes in occupance were measured in "structural units." A unit was defined for different occupance classes as: Residential, A-a single or double-family dwelling, B-a multi-family dwelling for 3-6 families, and C-a multi-family dwelling for more than 7 families; Commercial, Industrial and Transport, separate structures in multiples of 10,000 sq. ft, and open working space in multiples of 25,000 sq. ft; Public, each separate building.

In most places the commercial and industrial structures grew at a more rapid rate than residential structures, although in a few of the places such as in sectors of Los Angeles, Calif., the residential expansion was substantial. Public structures in most places showed substantial growth rates.

Certain areal patterns of change seem to repeat themselves in urban flood plains (Fig. 4). One typical pattern is found where residential areas in or bordering on the flood plain enlarge by moving down nearer the channel. A second, is that in which industries already established in a flood-hazard zone expand laterally along the stream bank and at right angles away from it. A third occurs where a central business district spreads out into the flood plain, following the major traffic arteries.

There is no readily discernable relation between rate of population growth and rate of change in structures on the flood plain: Too many other factors

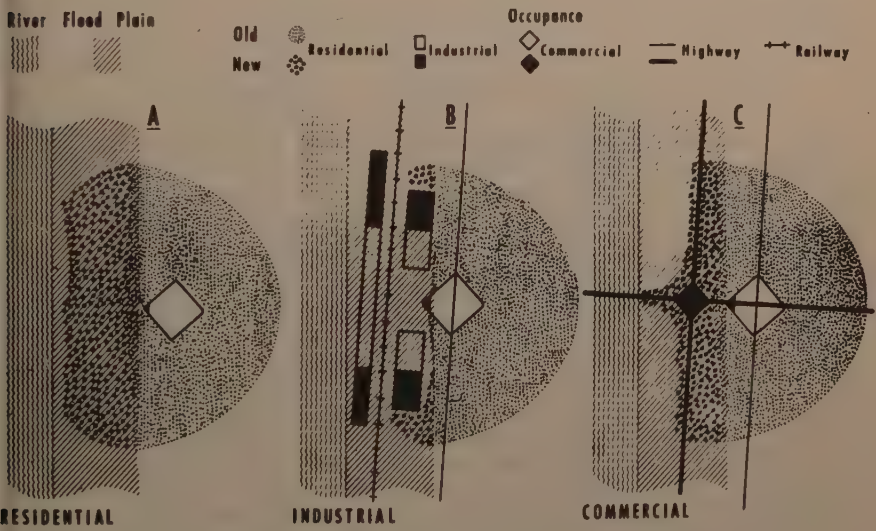

FIG. 4.—SCHEMATIC MAPS OF THREE STRATEGIC LAND-USE ASSOCIATIONS, SHOWING PATTERNS OF GROWTH IN THE FLOOD PLAIN.

such as urban function and availability of land play a part. It is clear, however, that two forms of public action have had a powerful stimulating effect upon invasion of flood plains. These are the highway program and the flood-control program. New highway construction in urban areas tends to follow the low gradients of stream valleys and the less densely settled sectors of some flood plains. It has caused substantial removal of low quality residential structures but it also has been an inducement for commercial and industrial establishments to follow along into the flood hazard zones.

As might be expected, the construction of new flood-protection works frequently has been the signal for accelerated movement into the flood plain. Thus, the completion of reliable works along the Trinity River in Dallas, Tex. assured development of a large-scale commercial district behind the levees.

But there are certain implications of flood-control works that are not as obvious. The lack of protection is not necessarily a deterrent to urban invasion of flood plains: In cities which have had no serious flood in 50 yr. as well as in cities that had unprecedented flooding only a few years before, there has been extensive building without any immediate prospect of protection. Moreover, in certain valleys, such as in the Tennessee at Chattanooga, the completion of river-control works upstream has been followed by further movement into the flood-plain where frequency has been reduced but where high flows still are possible. Although most Federal flood-control works are built to protect against a project flood and conceivably will one day, however infrequently, be exceeded by a larger flow, there is a universal disposition to believe that the rare flow will never come. This means that the number of situations in which a catastrophic disaster may follow one of those rare flows is increasing as new levees are completed.

These generalizations are illustrated in the near-by cities of Boulder and Denver.

FLOOD-PLAIN OCCUPANCE IN BOULDER

There has been no major flood in Boulder since 1894, when waters from an intense rainfall in the Rocky Mountain Front area of Boulder Creek inundated the areas to the north of the Creek as far as Water Street. (Fig. 5) In 1950 Congress authorized a combination levee, channel improvement, and bridge reconstruction project to protect sectors of the plain that had been built up over the years. The authorized project was to accommodate a flow equal to that of 1894, or about 12,000 cfs. A flow of as much as 15,000 cfs would not be impossible. The City decided not to take part in the authorized project which would have required a local contribution of $1,192,000 out of a total estimated cost of $1,707,000. So the situation stands today with no protection works, and with some local interests pushing for a Department of Agriculture watershed survey as a possible line to an alternative solution.

Since 1938, the area which was flooded has been the arena for vigorous new construction. Single-family residential structures have increased 34%, commerical structures 26%, and public structures 300%. The University of Colorado has built twenty eight new multiple-unit housing structures in the flood plain. The City has constructed its municipal building between the channel and the line of the proposed levee. The largest bank has its new building astride the proposed levee line. One modern hotel is under construction a short distance below the canyon mouth, and another along the creek between 24th and 28th streets.

Some managers of property in the flood plain believe that the area never will be flooded again. Others are taking a calculated risk, arguing that the benefits of daily use will more than offset the losses from an occasional overflow, and that another flood may not come in their period of occupance. They are correct in thinking that Boulder has not recently added to the national bill of flood losses. But when a flow equivalent to the one of 1894 does come down the channel—perhaps next year, perhaps not in our lifetime—the city will be on the front page of every newspaper in the country as a place where property losses exceeded several millions of dollars in a few hours. Hopefully, the pounding creek waters, laden with boulders and debris, may not cause loss of life. If the experience elsewhere applies here, that unpredictable but certain

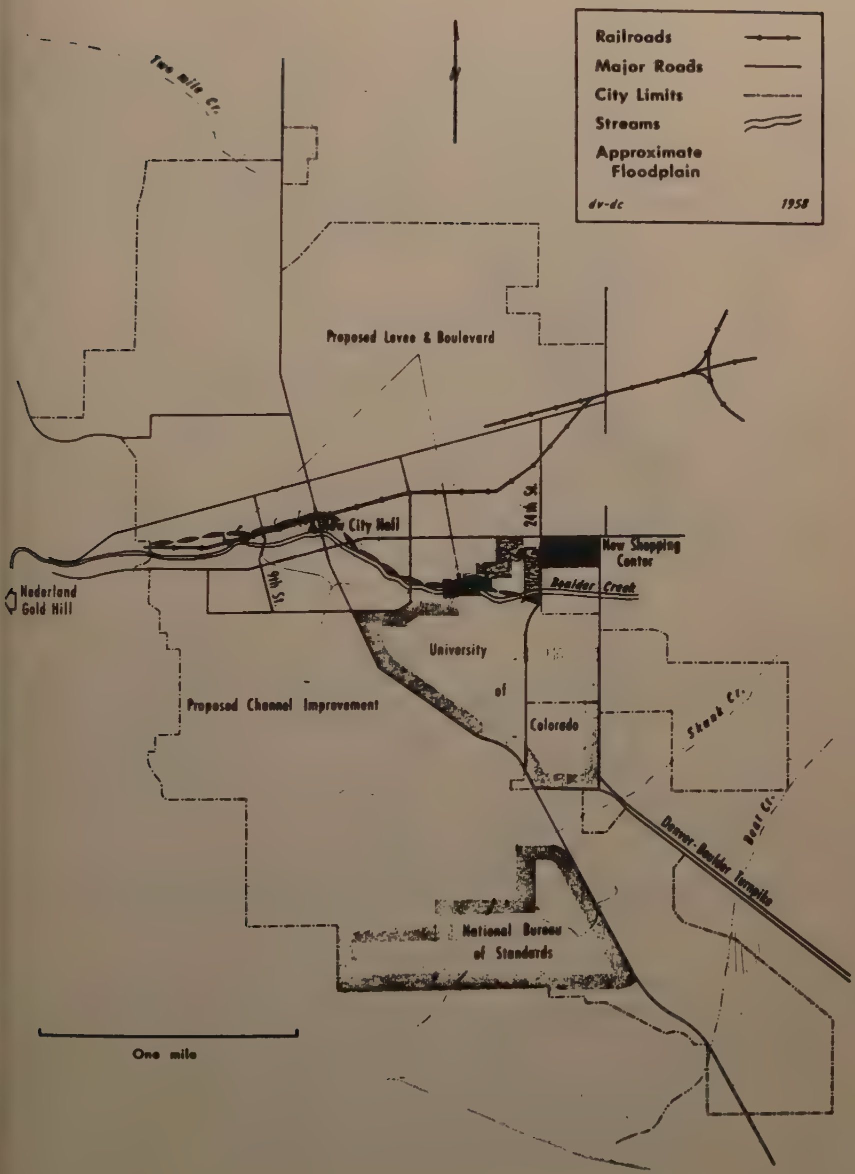

FIG. 5.—MAP OF BOULDER, COLO. SHOWING APPROXIMATE FLOOD PLAIN AND LOCATION OF SOME NEW DEVELOPMENTS.

flood will be followed by organization of an emergency citizens committee to obtain Federal flood control for Boulder.

FLOOD-PLAIN OCCUPANCE IN DENVER

Denver has three distinct flood situations, one of which has been largely solved, one of which daily becomes more acute, and one of which is in course of halting but imaginative solution. The Cherry Creek flood plain (Fig. 6) was the scene of a damaging overflow in 1933 that covered large sections of residential and commercial occupance in the city. This area now is protected by a Federal detention dam capable of controlling a storm runoff of more than 600 cfs per sq. mile. There has been argument that such protection is unduly

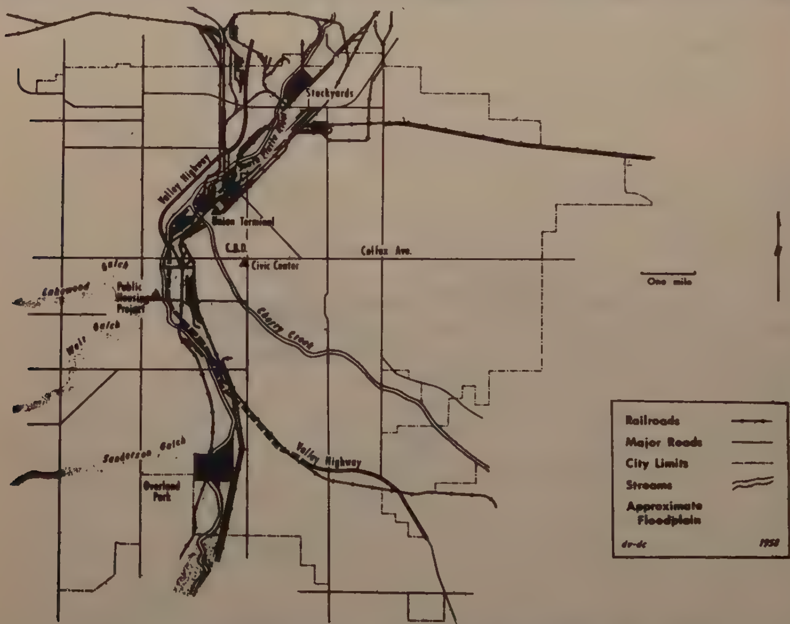

FIG. 6.—MAP OF DENVER, COLO. SHOWING APPROXIMATE FLOOD PLAIN AND LOCATION OF MAJOR TRANSPORTATION LINES.

expensive, but, regardless of cost, that sector of Denver seems to be off the rolls for flood-loss reporting.

Along the main stem of the South Platte River, both above and below the mouth of Cherry Creek, there is a broad plain that received partial protection from WPA works and that has not experienced flood flows exceeding the channel capacity of 13,000 cfs in recent years. The Congress, in 1950, authorized a reservoir and levee protection project to control flows of 85,500 cfs for this area. The dam was never started, and, indeed, could not be started today without dislodging a unit of the Martin Aircraft Corporation and a residential subdivision, both of which have since invaded the proposed reservoir area. Meanwhile, in the South Platte flood plain commercial structures have increased 26% since 1936, industrial structures 40% to 50%, and residential structures

slightly, notwithstanding extensive demolition for a new expressway in the valley. The damage potential unquestionably has increased at a rapid rate in this lower valley area.

A little progress has been made in the small tributary gulches where there has been a persistent tendency for residential and commercial occupance to press toward the dry stream beds, thus reducing the capacity of the channels and exposing more property to flood loss. Denver is one of the few cities that has begun a program of zoning, building regulation, and land acquisition to control further encroachment on such dry gulches, and a start has been made at metropolitan area planning for regulation across political boundaries.[7]

While the Cherry Creek area of flood loss has been eliminated by engineering works, the South Platte damage potential has been growing under the stimulation of highway and partial flood-control construction. A modest start is underway to curb further encroachment on the tributary gulches.

RANGE OF CHOICE IN ADJUSTMENT TO FLOOD HAZARD

The traditional approach to flood losses in the United States has been to either control the flood flows by engineering works or to bear the losses, letting the public come to the aid of the less prosperous flood sufferers. This is shown by the heavy public emphasis upon investment in Federal flood-control works and by the Red Cross and other emergency aid programs.

There are, of course, other possible adjustments which individual property managers can make. The whole range of choice may be listed as follows:

Bear the Losses.—A good many households and firms bear the losses when they occur, and a few actually make financial plans to do so.

Emergency Evacuation and Rescheduling.—Property may be removed from the reach of floods, and property movement and production operations may be rescheduled so as to avoid losses through interruption. This requires a relatively accurate system of flood forecasting and a plan for emergency action when the critical forecast is received.

Prevent Flows.—In some areas it is practicable to prevent certain flows, particularly the more frequent flows, by land treatment and associated measures. The experience with such measures is only beginning to build up.

Elevate Land.—By land fill it is practicable in some situations to raise property above the level of flood waters. The effect upon channel capacity varies according to location with respect to flowage and pondage areas on the flood plain.

Control Flows.—This is the conventional engineering solution, involving channel improvement, levees, cut-offs, and storage or detention dams.

Change Structures.—In advance of a flood warning, structures may be altered so as to prevent or reduce flood losses when the water rises. These changes may include such alterations as packing of machinery, bricking in of low openings, cut-off valves on sewers, and rearrangement of electrical circuits.

Change Land Use.—The use of flood-plain land may be changed so as to introduce a use that is less susceptible to flood loss. This may range from the

7 "Report for Inter-county Regional Planning Commission on Storm Drainage in the Denver Metropolitan Area," by Dale Rea, 1957.

transfer of an entire town from a riverine to an upland site, to public acquisition of flood-hazard areas for recreational or parking purposes. It may be guided by public regulation, including zoning, building ordinances, subdivision regulation, and land acquisition.

Insure.—Although insurance against flood losses is not generally available in the United States, there is an inactive program for Federal-state subsidized insurance under the Flood Insurance Act of 1956, and there are a few instances of coverage by private companies where special structural adjustments have been made.

Public Relief.—Through the medium of the Red Cross or by direct grants and loans under Public Roads, Civil Defense, Corps of Engineers and small business programs, public aid is given to individuals and local governments that suffer heavily from floods.

There are concrete examples of each type of adjustment, and there are strong arguments that can be made for and against each type of adjustment, depending upon the local situation. To understand how and why particular adjustments are made in a given situation it is necessary to look to the various elements that enter into decisions as to management of urban flood-plain properties.

ELEMENTS IN DECISIONS AS TO FLOOD-PLAIN USE

As with most other resource-management situations, at least seven considerations seem to enter into decisions as to flood-plain adjustments. It may be helpful to examine how each one figures in the urban situations which have been studied.

Estimating the Resource.—There is widespread ignorance of the flood hazard and a tendency to minimize it. Many people building or buying in flood plains are unaware of the precise hazard they are running, or grossly misinterpret the technical estimates. This applies even in places where there have been public plans for flood protection. A man says he need not worry about floods because a 200-yr. flood occurred the year before. A Federal housing insurance office says it does not insure mortgages for new buildings in flood plains but has no map showing where the flood-hazard areas are located.

Discounting Future Benefits and Costs.—It is common in Federal flood prevention and flood-control studies to discount future benefits and costs from the proposed works. These cost-benefit ratios have significance in Congressional decisions so long as they are below unity. There is a tendency to group all programs with a ratio of more than unity together and to pay little attention to the means by which the ratios were calculated. So far as private planning of flood-plain adjustments are concerned there is little evidence of such discounting procedure. A few managers of large industrial units make cost-benefit calculations but most managers do not.

Harmonizing Two or More Uses.—In the Federal plans there is a disposition to harmonize flood-control plans with other water uses such as irrigation and navigation. At the level of local governments and private managers such multiple-uses tend to be ignored. Few attempts are made to prevent harmful encroachments, and there is little attention to combining levees with highways, or to harmonizing a park development with a floodway improvement. One of the hopeful moves in this direction is under planning supported by the Urban Renewal Administration.

Projecting Future Demand.—Typically, the Federal agencies assume that here will be little change in the demand for flood-plain land while private managers tend to overestimate the effect of protection and to rush in at the hint r prospect of some control work.

Projecting Technological Change.—Private managers tend to overestimate he physical benefits from any kind of prevention or protection work. The ompletion of a single-purpose power dam upstream, (and even downstream), he beginning of a watershed improvement, or the clearing of an upstream hannel is taken as assurance that floods will be abated in the future. Thus, a ittle protection work may encourage a great deal of channel encroachment.

Integrating Regional Uses.—While the Federal agencies try to assess the onsequences of a flood-control program upon other water uses in the same or djoining basins, there is little attention to this on the part of local and private managers, and metropolitan area plans for storm water disposal are rare. ven less common are efforts to assess the possible use of flood plains in erving regional requirements for land and for riverine facilities.

Setting Social Guides.—We come now to one element which affects all the est. Society, through its public agencies, definitely restricts some kinds of ecisions by property managers and clearly encourages some other kinds of ction. Federal agencies tend to guide the occupance of flood plains by providing prevention and protection plans, by giving public relief, by issuing storm nd flood warnings, and by offering information on flood occurrence. Seven tates attempt to regulate channel encroachment. A few cities and counties xercise some regulation over flood-plain use. One Federal agency cooperates ith state planning agencies in assisting with local plans for flood-plain use.[8]

BROADENING THE RANGE OF CHOICE

In the present circumstances the social guides in the form of information, egulation, and investment affecting flood-plain occupance tend to encourage urther encroachment upon the flood plains at the same time that they lead to eavier Federal expenditures for flood control. It is not entirely whimsical to ay that the national situation is somewhat like a local situation where investigation showed forty new houses in a flooded area could not be economically rotected by levee works. It was noted, however, that if twenty additional ouses were to be constructed protection then would be feasible. Given the xisting policies one may expect that the additional twenty houses will in time e built and that Federal largess will follow in due course.

We know that the managers of flood-plain properties often underestimate the azard, fail to discount future benefits and costs, overestimate the demand for eir land, overestimate the effects of engineering and land-treatment works, nd ignore possible combinations of flood-loss reduction with other local improvements or with regional plans for land and water use. We also know that s a practical matter most managers who face a flood threat have a choice beeen only two alternatives. They can bear the losses, or they can press for Federal project of some sort. They do not receive technical advice as to the ossibilities of emergency evacuation, land elevation, structural changes, or

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8 "Regulating Flood-Pain Development," by Francis C. Murphy, Univ. of Chicago eorgraphy Research Paper, 1958, No. 56.

land-use changes. They do not have insurance readily available. If they suffe unduly heavy losses they have the prospect of public aid. If they are energeti enough they may receive public protection. With few exceptions they do nc know the precise character of the flood hazard, and they are not subject to pub lic curbs against any further encroachment upon the stream channel.

If we are to break out of the present situation in which flood losses promis to keep pace with flood protection for at least several decades it seems essen tial to broaden the range of choice. Each manager—public or private—shoul be given the opportunity to choose among the whole range of possible adjust ments, with the public agencies strictly limiting those choices which woul cause damage to others or to the public safety. This calls for a new and fres approach to the problem of flood loss reduction in the United States. An impor tant beginning in that direction was made at the Conference called by the Coun cil of State Governments in December, 1958.[9] The new approach does not re quire abandonment of existing public programs. It requires that they b supplemented by new or expanded efforts to broaden the choice open to all wh face flood hazard.

The chief directions in which it now seems necessary to move are these:

1. Publication and wide distribution of flood hazard maps and reports for a important flood hazard areas, to be made available to both property manager and to public lending and construction agencies.

2. Technical advice to property managers as to the means and costs of re ducing flood losses by emergency measures, by structural changes, and b land-use changes.

3. A cooperative Federal-state program of flood insurance in which pre miums are in proportion to hazard.

4. State regulation of any further channel encroachments which would caus damage to others or to the public safety, and Federal requirement that addi tional Federal expenditures for flood control be contingent upon such regula tion.

5. An improved system for issuing flood warnings and for getting them int the hands of the property manager concerned.

6. Technical assistance to local governments in drawing up plans for re ducing flood losses by whatever combination of means may seem most feas ible.

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9 "A New Attack on Flood Losses," by Gilbert F. White, State Government, Sprin 1959.

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Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION

(Over)

APPLICATION OF SNOW HYDROLOGY TO THE COLUMBIA BASIN[a]

Closure by O. A. Johnson and P. B. Boyer

O. A. JOHNSON, F. ASCE[1] and P. B. BOYER.[2]—Mr. Miller has presented ı well-written, thoughtful discussion, adding several explanations for which he writers are grateful. They agree that all the detailed exploratory studies omprising a major research effort should be made available to those interested in the general subject covered. His statement that the original papers re nowhere referred to is a little misleading, in that the referenced "Snow Hydrology" has some 14 pages filled with references to works by writers in he general field of hydrology as well as those engaged in analyzing and reorting upon the results of the recent laboratory measurements. Some of hese were written by Mr. Miller.

The water-balance approach to forecasting volume of streamflow from nelting snow may indeed replace, in time, the index methods now used. However, much better coverage must first be established of such elements as now water equivalent, soil and snow priming, evaporation and other losses, nd precipitation. Until such time, the index methods presently in use must ontinue to serve. In fact, until long-term weather forecasts can be made vith improved reliability, forecasts of spring snowmelt runoff will continue to e made with considerable uncertainty.

Mr. Miller chooses to reverse the order of consideration of four equations oncerning the heat balance concept, going from a formula containing a large umber of terms to one containing only two temperature measurements. For resentation to a hydrologist completely familiar with the subject, this order f consideration might logically be followed, but the writers assumed that ome persons whose knowledge of snow hydrology was limited would read the aper. In this case, the usual and preferable method, where a choice exists, s to start with the least-complicated situation and to develop formulas that onsider additional parameters, until the most complicated situation is covred.

Mr. Miller emphasizes the sparsity of radiation observations, particularly r long wave radiation. Several relatively inexpensive and self-contained intruments for measuring and recording solar radiation have been developed, ne by an English concern, based upon research by the Meteorological Office. s a result, it may soon be possible to equip stations in mountainous basins ith instruments for measuring daily solar radiation; this information would e reported by the observer to the using offices by electric means in order at better forecasts of runoff may be made.

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[a] January, 1959, by Oliver A. Johnson and Peter B. Boyer.

[1] Hydr. Engr., North Pacific Div., Corps of Engrs., U. S. Dept. of the Army, Portland, reg.

[2] Hydr. Engr., Portland Dist., Corps of Engrs., U. S. Dept. of the Army, Portland, reg.

DETERMINATION OF HYDROLOGIC FREQUENCY FACTORS[a]

Discussion by William H. Sammons

WILLIAM H. SAMMONS,[1] M. ASCE.—The writer is deeply indebted to the author for personal assistance prior to publication of this graphical method for the determination of K-values.[2]

In April 1958, the writer was "knee-deep" in the preparation of a paper[3] for the May meeting of the American Geophysical Union. A phone call to the author solved the major problem which dictated a solution. This will be referred to later as Method A and will be presented briefly for the benefit of the reader. Methods employed in the May, 1958 paper stimulated the writing of a subsequent paper[4] in September, 1958. This latter paper presented theory, examples and K-values complied from other sources.[5,6,7,8,9,10] (A few new applications were also presented.

The K-values presented here as Table 2, "Theoretical Log-Probability Critical Values," will be discussed under Method B, and was designed to supplement available tables.[4,8]

Method A.—Limited time demanded the expediency of this procedure which is the author's suggested method (with a few liberties taken). The writer believes the K-values developed (tables of the K-value were furnished by the author and a few office copies were printed) served the purpose for which they were intended.[3] The writer does not recommend the indiscriminate use of the unpublished table. Briefly, Method A is as follows:

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[a] July, 1959, by Ven Te Chow.

[1] Hydraulic Engineer, U. S. Dept. of Agric., Soil Conservation Service, Central Technical Unit, Beltsville, Maryland.

[2] "A Graphical Method for Determining the Log-Normal Hydrological Frequency Factor," by Ven Te Chow, (Private communication with Ven Te Chow, April 5, 1958. Not for publication).

[3] "Agricultural Watershed Influence on Runoff Potential," by Melvin H. Kleen, and Wm. H. Sammons, presented at the thirty-ninth annual meeting of the May 5-8, 1958 at Washington, D. C. (Manuscript only).

[4] "Relationship Between the Original and Transformed Variates for the Logarithmic-Probability Law," by Wm. H. Sammons, (For in-Service use and distribution at the SCS Hydrology Meeting, Jackson, Miss., October 13-17, 1958).

[5] "On the Logarithmic-Probability Law," by A. A. Kalinske, Trans. AGU, 1946, Vol. 27, No. 5, pp. 709-710.

[6] Reference 5 of Proc. Paper 2084, by Ven Te Chow.

[7] "A Nomogram for Log-Normal Frequency Analysis," by L. L. Weiss, Trans. AGU, 1957, Vol. 38, No. 1, pp. 33-37.

[8] "Critical Values of the Log-Normal Distribution," by Jack Moshman, J. A. Statistical Assoc., 1953, Vol. 48, No. 263, pp. 600-609.

[9] "The Log-Normal Distribution," by J. Aitchison and J. A. C. Brown, Cambridge Univ. Press, 1957.

[10] "Fitting a Generalized Log-Normal Distribution to Hydrologic Data," by D. L. Brakensiek, Trans. AGU, 1958, Vol. 39, No. 3, pp. 469-473.

1. The author's procedure was followed verbatim.
2. Figures were plotted similar to the author's Fig. 1.
3. Probability lines were developed for twelve "key" C_V-values (C_V = 1.1171 - 3.455).

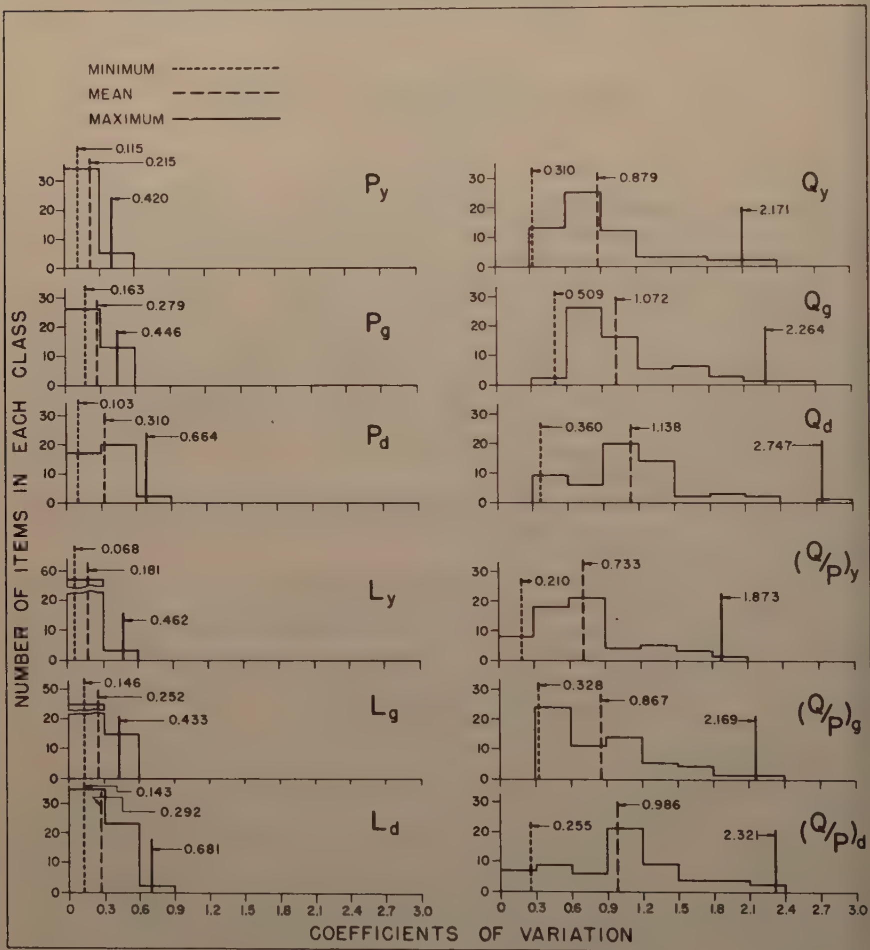

FIG. 1.—DISTRIBUTION OF THE COEFFICIENTS OF VARIATION.

4. K-values (graphical determinations) were plotted on the logarithmic scale versus C_V-values on the arithmetic scale of semi-logarithmetic graph paper.
5. K-values for nine levels of probability used in Table 2 of Reference 6 were read from these graphs as needed for use in Reference 3.
6. These nine K-values for specific C_V-values were smoothed by computing $X/\overline{X}$; plotting on probability paper; drawing an average line by eye; re-computing K-values by the author's Eq. 1.

7. Smoothed K-values were used to calculate X-magnitudes of the hydrologic event.
8. A table of approximately 100 C_s (or C_v) values were tabulated with K-values at the nine levels of probability.

Fig. 1 is presented to illustrate the extent to which this unpublished tabulation of K-values was employed.[3]

Method B.—K-values developed by Method B could have utilized an IBM-650 computer to a great advantage and under a different set of prevailing circumstances, it would probably have been worth while as an official project. Briefly, Method B is as follows:

1. The author's Eq. 9 was modified as follows:

$$X_k = M \exp K \sigma_y \quad \text{(9')}$$

where K-values for $C_v = 0.0000$ and a few selected values are tabulated as Table 1.

TABLE 1.—THEORETICAL LOG-PROBABILITY CRITICAL VALUES FOR THE STANDARD NORMAL VARIATE EXCEEDED WITH SPECIFIED PROBABILITY[a]

Probability in %	K-Value	Probability in %	K-Value
99.9	-3.090232	0.10	+3.090232
99.75	2.807034	0.25	2.807034
99.6	2.652070	0.40	2.652070
99.5	2.5758	0.50	2.5758
99	2.326348	1.00	2.326348
97.5	-1.959964	2.50	+1.959964
96	-1.750868	4	+1.750868
95	1.6449	5	1.6449
93.5	1.514102	6.5	1.514102
90	1.281552	10	1.281552
85	1.036433	15	1.036433
75	-0.674490	25	+0.674490
50	0.000000	50	0.000000

[a] Probability in percentage equal to or greater than the given variate.

2. Five levels of probability were employed to compute K-values.
3. Codex Book Company, Inc., Logarithmic Normal Probability Paper No. 31,376 was used to plot $X/\bar{X}$ versus probability at the 99, 84.13, 50, 15.87 and 1% probability levels (used in smoothing).
4. The five levels of probability, "computed" K-values, were plotted on K and E, 20 x 20 to the inch, coordinate paper No. 359-10 1/2 LG, versus C_s.
5. Smooth curves were drawn and computational errors located and corrected by computational methods.
6. The five K-values for specific C_s (or C_v)-values were smoothed by the process described under Method A, step 6. These were later plotted on

TABLE 2.—THEORETICAL LOG-PROBABILITY CRITICAL VALUES

Coefficients of			Probability, in percentage, equal to or greater than the given variate									
Skew C_s	Variation C_v	ΔC_v	99 -	ΔK	84.13 -	ΔK	50 -	ΔK	15.87 +	ΔK	1 +	ΔK
3.00	0.8177	--	1.0437	--	.76004	--	.27621	--	.71333	--	3.7787	--
3.01	0.8197	20	1.0420	17	.75929	75	.27648	27	.71235	98	3.7804	17
3.02	0.8217	20	1.0402	18	.75855	74	.27675	27	.71139	96	3.7822	18
3.03	0.8237	20	1.0384	18	.75780	75	.27701	26	.71043	96	3.7840	18
3.04	0.8257	20	1.0366	18	.75696	84	.27718	17	.70948	95	3.7863	23
3.05	0.8277	20	1.0348	18	.75622	74	.27745	27	.70853	95	3.7882	19
3.06	0.8296	19	1.0331	17	.75544	78	.27765	20	.70748	105	3.7895	13
3.07	0.8316	20	1.0313	18	.75470	74	.27790	25	.70654	94	3.7913	18
3.08	0.8336	20	1.0296	17	.75396	74	.27816	26	.70561	93	3.7932	19
3.09	0.8356	20	1.0278	18	.75317	79	.27841	25	.70449	112	3.7946	14
3.10	0.8375	19	1.0261	17	.75244	73	.27860	19	.70365	84	3.7965	19
3.11	0.8395	20	1.0244	17	.75170	74	.27885	25	.70273	92	3.7980	15
3.12	0.8414	19	1.0227	17	.75101	69	.27913	28	.70171	102	3.7998	18
3.13	0.8434	20	1.0209	18	.75019	82	.27929	16	.70081	90	3.8014	16
3.14	0.8453	19	1.0193	16	.74946	73	.27956	27	.69961	120	3.8030	16
3.15	0.8473	20	1.0176	17	.74873	73	.27981	25	.69871	90	3.8046	16
3.16	0.8492	19	1.0159	17	.74800	73	.27999	18	.69789	82	3.8062	16
3.17	0.8511	19	1.0142	17	.74727	73	.28017	18	.69690	99	3.8087	16
3.18	0.8531	20	1.0126	16	.74650	77	.28040	23	.69601	89	3.8094	16
3.19	0.8550	19	1.0109	17	.74573	77	.28058	18	.69502	99	3.8110	16
3.20	0.8569	19	1.0093	16	.74505	68	.28084	26	.69404	98	3.8125	15
3.21	0.8588	19	1.0077	16	.74432	73	.28102	18	.69325	79	3.8140	15
3.22	0.8608	20	1.0060	17	.74356	76	.28125	23	.69219	106	3.8156	16
3.23	0.8627	19	1.0044	16	.74288	68	.28150	25	.69123	96	3.8172	16
3.24	0.8646	19	1.0027	17	.74212	76	.28167	17	.69026	97	3.8184	12
3.25	0.8665	19	1.00113	160	.74140	72	.28184	17	.68948	78	3.8199	15
3.26	0.8684	19	.99953	160	.74068	72	.28210	26	.68834	114	3.8214	15
3.27	0.8703	19	.99792	161	.73996	72	.28226	16	.68757	77	3.8229	15
3.28	0.8722	19	.99631	161	.73921	75	.28242	16	.68662	95	3.8242	13
3.29	0.8741	19	.99475	156	.73854	67	.28267	25	.68568	94	3.8258	16
3.30	0.8760	19	.99318	157	.73786	68	.28292	25	.68474	94	3.8273	15
3.31	0.8778	18	.99166	152	.73712	74	.28302	10	.68388	86	3.8289	16
3.32	0.8797	19	.99009	157	.73645	67	.28326	24	.68295	93	3.8300	11
3.33	0.8816	19	.98851	158	.73570	75	.28342	16	.68203	92	3.8317	17
3.34	0.8835	19	.98695	156	.73503	67	.28366	24	.68110	93	3.8329	12
3.35	0.8854	19	.98537	158	.73428	75	.28381	15	.68019	91	3.8344	15
3.36	0.8872	18	.98380	157	.73362	66	.28400	19	.67935	84	3.8359	15
3.37	0.8891	19	.98234	146	.73287	75	.28415	15	.67844	91	3.8374	15
3.38	0.8909	18	.98086	148	.73221	66	.28433	18	.67761	83	3.8386	12
3.39	0.8928	19	.97933	153	.73151	70	.28456	23	.67652	109	3.8399	13
3.40	0.8946	18	.97780	153	.73085	66	.28474	18	.67570	82	3.8411	12
3.41	0.8965	19	.97630	150	.73011	74	.28488	14	.67480	90	3.8426	15
3.42	0.8983	18	.97485	145	.72946	65	.28505	17	.67399	81	3.8438	12
3.43	0.9002	19	.97333	152	.72876	70	.28528	23	.67292	107	3.8451	13
3.44	0.9020	18	.97188	145	.72811	65	.28545	17	.67211	81	3.8464	13
3.45	0.9039	19	.97042	146	.72737	74	.28559	14	.67123	88	3.8476	12
3.46	0.9057	18	.96895	147	.72672	65	.28576	17	.67043	80	3.8489	13
3.47	0.9075	18	.96748	146	.72603	69	.28593	17	.66946	97	3.8502	13
3.48	0.9093	18	.96603	145	.72538	65	.28609	16	.66866	80	3.8514	12
3.49	0.9112	19	.96452	151	.72465	73	.28623	14	.66780	86	3.8526	12
3.50	0.9130	18	.96310	142	.72397	68	.28639	16	.66683	97	3.8539	13
3.51	0.9148	18	.96166	144	.72332	65	.28655	16	.66605	78	3.8552	13
3.52	0.9166	18	.96024	142	.72264	68	.28672	17	.66509	96	3.8565	13
3.53	0.9184	18	.95883	141	.72199	65	.28688	16	.66431	78	3.8576	11
3.54	0.9202	18	.95741	142	.72131	68	.28703	15	.66336	95	3.8589	13
3.55	0.9220	18	.95600	141	.72067	64	.28719	16	.66259	77	3.8601	12

TABLE 2.—THEORETICAL LOG-PROBABILITY CRITICAL VALUES

Coefficients of			Probability, in percentage, equal to or greater than the given variate									
Skew	Variation		99	ΔK	84.13	ΔK	50	ΔK	15.87	ΔK	1	ΔK
C_s	C_v	ΔC_v	-		-		-		+		+	
3.55	0.9220	18	.95600	141	.72067	64	.28719	16	.66259	77	3.8601	12
3.56	0.9238	18	.95459	141	.71999	68	.28735	16	.66165	94	3.8614	13
3.57	0.9256	18	.95319	140	.71931	68	.28750	15	.66071	94	3.8625	11
3.58	0.9274	18	.95180	139	.71867	64	.28766	16	.65995	76	3.8637	12
3.59	0.9292	18	.95040	140	.71800	67	.28781	15	.65902	93	3.8649	12
3.60	0.9310	18	.94901	139	.71736	64	.28796	15	.65827	75	3.8660	11
3.65	0.9399	89	.94215	686	.71408	328	.28865	69	.65409	418	3.8712	52
3.70	0.9487	88	.93547	668	.71087	321	.28935	70	.64990	419	3.8765	53
3.75	0.9574	87	.92895	652	.70773	314	.29004	69	.64568	422	3.8818	53
3.80	0.9661	87	.92249	646	.70455	318	.29064	60	.64154	414	3.8868	50
3.85	0.9747	86	.91619	630	.70146	309	.29124	60	.63755	399	3.8916	48
3.90	0.9832	85	.91004	615	.69844	302	.29185	61	.63352	403	3.8962	46
3.95	0.9916	84	.90401	603	.69544	300	.29239	54	.62963	389	3.9004	42
4.00	1.0000	84	.89806	595	.69248	296	.29291	52	.62580	383	3.9045	41
4.05	1.0083	83	.89225	581	.68951	297	.29337	46	.62194	386	3.9086	41
4.10	1.0165	82	.88656	569	.68663	288	.29384	47	.61820	374	3.9125	39
4.15	1.0247	82	.88083	573	.68363	300	.29429	45	.61450	370	3.9162	37
4.20	1.0328	81	.87545	538	.68095	268	.29475	46	.61065	385	3.9200	38
4.25	1.0408	80	.87008	537	.67819	276	.29515	40	.60706	359	3.9234	34
4.30	1.0488	80	.86477	531	.67545	274	.29553	38	.60352	354	3.9266	32
4.35	1.0567	79	.85957	520	.67270	275	.29587	34	.59993	359	3.9297	31
4.40	1.0645	78	.85449	508	.67005	265	.29621	34	.59646	347	3.9327	30
4.45	1.0723	78	.84947	502	.66741	264	.29654	33	.59304	342	3.9356	29
4.50	1.0800	77	.84457	490	.66478	263	.29682	28	.58956	348	3.9383	27
4.55	1.0877	77	.83971	486	.66221	257	.29715	33	.58614	342	3.9408	25
4.60	1.0953	76	.83497	474	.65968	253	.29744	29	.58282	332	3.9433	25
4.65	1.1029	76	.83026	471	.65711	257	.29764	20	.57955	327	3.9457	24
4.70	1.1104	75	.82567	459	.65465	246	.29793	29	.57623	332	3.9480	23
4.75	1.1178	74	.82117	450	.65219	246	.29811	18	.57315	308	3.9503	23
4.80	1.1252	74	.81673	444	.64977	242	.29833	22	.56996	319	3.9523	20
4.85	1.1325	73	.81239	434	.64740	237	.29858	25	.56672	324	3.9542	19
4.90	1.1398	73	.80808	431	.64501	239	.29875	17	.56352	320	3.9559	17
4.95	1.1470	72	.80387	421	.64266	235	.29889	14	.56056	296	3.9575	16
5.00	1.1542	72	.79970	417	.64035	231	.29907	18	.55749	307	3.9591	16
5.10	1.1684	142	.79159	811	.63583	452	.29934	27	.55156	593	3.9622	31
5.20	1.1824	140	.78373	786	.63137	446	.29958	24	.54572	584	3.9650	28
5.30	1.1962	138	.77612	771	.62705	432	.29978	20	.54011	561	3.9671	21
5.40	1.2098	136	.76875	737	.62281	424	.29995	17	.53445	566	3.9689	18
5.50	1.2232	134	.76161	714	.61869	412	.30009	14	.52900	545	3.9706	17
5.60	1.2365	133	.75463	698	.61463	406	.30018	9	.52371	529	3.9718	12
5.70	1.2496	131	.74788	675	.61067	396	.30024	6	.51848	523	3.9730	12
5.80	1.2625	129	.74133	655	.60682	385	.30028	4	.51345	503	3.9739	9
5.90	1.2753	128	.73494	639	.60302	380	.30028	0	.50855	490	3.9746	7
6.00	1.2879	126	.72873	621	.59932	370	.30026	2	.50371	484	3.9750	4
6.10	1.3004	125	.72267	606	.59567	365	.30024	2	.49874	497	3.9752	2
6.20	1.3127	123	.71680	587	.59212	355	.30016	8	.49419	455	3.9753	1
6.30	1.3249	122	.71106	574	.58863	349	.30008	8	.48964	455	3.9751	2
6.40	1.3369	120	.70549	557	.58522	341	.29995	13	.48524	440	3.9749	2
6.50	1.3488	119	.70006	543	.58187	335	.29983	12	.48084	440	3.9744	5
6.60	1.3605	117	.69478	528	.57861	326	.29969	14	.47658	426	3.9738	6
6.70	1.3722	117	.68957	521	.57537	324	.29955	14	.47228	430	3.9730	8
6.80	1.3837	115	.58453	504	.57223	314	.29939	16	.46812	416	3.9720	10
6.90	1.3950	113	.67964	489	.56913	310	.29919	20	.46409	403	3.9709	11
7.00	1.4063	113	.67481	483	.56611	302	.29902	17	.46012	397	3.9697	12
7.10	1.4174	111	.67012	469	.56313	298	.29879	23	.45629	383	3.9684	13

TABLE 2.—THEORETICAL LOG-PROBABILITY CRITICAL VALUES (Continued)

Coefficients of			Probability, in percentage, equal to or greater than the given variate									
Skew C_s	Variation C_v	ΔC_v	99 −	ΔK	84.13 −	ΔK	50 −	ΔK	15.87 +	ΔK	1 +	ΔK
7.20	1.4284	110	.66554	458	.56022	291	.29859	20	.45243	386	3.9670	14
7.30	1.4394	110	.66102	452	.55732	290	.29833	26	.44874	369	3.9656	14
7.40	1.4501	107	.65667	435	.55453	279	.29813	20	.44498	376	3.9640	16
7.50	1.4608	107	.65238	429	.55175	278	.29785	28	.44149	349	3.9623	17
7.60	1.4718	106	.64817	421	.54904	271	.29762	23	.43787	362	3.9603	20
7.70	1.4819	105	.64407	410	.54635	269	.29734	28	.43438	349	3.9585	18
7.70	1.4819	105	.64407	410	.54635	269	.29734	28	.43438	349	3.9585	18
7.80	1.4923	104	.64003	404	.54373	262	.29709	25	.43096	342	3.9564	21
7.90	1.5026	103	.63608	395	.54114	259	.29679	30	.42768	328	3.9543	21
8.00	1.5127	101	.63226	382	.53862	252	.29652	27	.42428	340	3.9521	22
8.10	1.5228	101	.62848	378	.53613	249	.29622	30	.42115	313	3.9499	22
8.20	1.5328	100	.62479	369	.53368	245	.29593	29	.41797	318	3.9476	23
8.30	1.5427	99	.62115	364	.53127	241	.29562	31	.41486	311	3.9453	23
8.40	1.5526	99	.61757	358	.52886	241	.29531	31	.41169	317	3.9430	23
8.50	1.5623	97	.61409	348	.52655	231	.29503	28	.40861	308	3.9407	23
8.60	1.5719	96	.61068	341	.52425	230	.29470	33	.40569	292	3.9384	23
8.70	1.5815	96	.60731	337	.52198	227	.29437	33	.40282	287	3.9359	25
8.80	1.5910	95	.60401	330	.51975	223	.29405	32	.39990	292	3.9335	24
8.90	1.6004	94	.60079	322	.51757	218	.29375	30	.39704	286	3.9310	25
9.00	1.6097	93	.59760	319	.51541	216	.29340	35	.39434	270	3.9284	26
9.10	1.6189	92	.59451	309	.51328	213	.29307	33	.39159	275	3.9259	25
9.20	1.6281	92	.59143	308	.51119	209	.29276	31	.38878	281	3.9233	26
9.30	1.6372	91	.58842	301	.50913	206	.29241	35	.38622	256	3.9205	28
9.40	1.6462	90	.58549	293	.50710	203	.29208	33	.38361	261	3.9178	27
9.50	1.6552	90	.58256	293	.50509	201	.29174	34	.38103	258	3.9151	27
9.60	1.6640	88	.57974	282	.50313	196	.29141	33	.37853	250	3.9123	28
9.70	1.6729	89	.57688	286	.50116	197	.29106	35	.37603	250	3.9095	28
9.80	1.6816	87	.57414	274	.49925	191	.29071	35	.37360	243	3.9067	28
9.90	1.6903	87	.57143	271	.49736	189	.29039	32	.37110	250	3.9040	27
10.00	1.6989	86	.56875	268	.49549	187	.29003	36	.36874	236	3.9010	30
10.10	1.7074	85	.56615	260	.49366	183	.28968	35	.36643	231	3.8980	30
10.20	1.7159	85	.56355	260	.49183	183	.28933	35	.36404	239	3.8951	29
10.30	1.7243	84	.56101	254	.49006	177	.28900	33	.36180	224	3.8921	30
10.40	1.7327	84	.55851	250	.48827	179	.28864	36	.35948	232	3.8890	31
10.50	1.7410	83	.55604	247	.48654	173	.28830	34	.35730	218	3.8860	30
10.60	1.7493	83	.55359	245	.48480	174	.28795	35	.35505	225	3.8830	30
10.70	1.7574	81	.55123	236	.48313	167	.28761	34	.35295	210	3.8800	30
10.80	1.7655	81	.54887	236	.48146	167	.28727	34	.35079	216	3.8769	31
10.90	1.7736	81	.54654	233	.47979	167	.28690	37	.34873	206	3.8738	31
11.00	1.7816	80	.54427	227	.47816	163	.28655	35	.34671	202	3.8707	31
11.10	1.7896	80	.54199	228	.47654	162	.28622	33	.34460	211	3.8676	31
11.20	1.7975	79	.53978	221	.47495	159	.28586	36	.34256	204	3.8646	30
11.30	1.8053	78	.53760	218	.47339	156	.28551	35	.34063	193	3.8615	31
11.40	1.8131	78	.53546	214	.47183	156	.28516	35	.33863	200	3.8584	31
11.50	1.8209	78	.53331	215	.47028	155	.28482	34	.33665	198	3.8552	32
11.60	1.8286	77	.53121	210	.46877	151	.28448	34	.33470	195	3.8522	30
11.70	1.8362	76	.52916	205	.46728	149	.28414	34	.33279	191	3.8491	31
11.80	1.8439	77	.52710	206	.46578	150	.28378	36	.33096	183	3.8460	31
11.90	1.8514	75	.51510	200	.46433	145	.28343	35	.32919	177	3.8429	31
12.00	1.8589	75	.52312	198	.46289	144	.28310	33	.32726	193	3.8398	31
12.10	1.8663	74	.52117	195	.46148	141	.28277	33	.32553	173	3.8368	30
12.20	1.8738	75	.51921	196	.46004	144	.28242	35	.32371	182	3.8337	31
12.30	1.8811	73	.51733	188	.45866	138	.28207	35	.32203	168	3.8306	31
12.40	1.8884	73	.51546	187	.45728	138	.28172	35	.32027	176	3.8274	32
12.50	1.8957	73	.51358	188	.45591	137	.28138	34	.31862	165	3.8243	31
12.60	1.9030	73	.51174	184	.45455	136	.28103	35	.31689	173	3.8213	30

TABLE 2.—THEORETICAL LOG-PROBABILITY CRITICAL VALUES (Continued)

Coefficients of			Probability, in percentage, equal to or greater than the given variate									
Skew	Variation		99		84.13		50		15.87		1	
C_s	C_v	ΔC_v	-	ΔK	-	ΔK	-	ΔK	+	ΔK	+	ΔK
12.70	1.9102	72	.50993	181	.45321	134	.28070	33	.31519	170	3.8182	31
12.80	1.9173	71	.50815	178	.45191	130	.28036	34	.31361	158	3.8151	31
12.90	1.9244	71	.50626	189	.45047	144	.27995	41	.31187	174	3.8120	31
13.00	1.9315	71	.50463	163	.44930	117	.27969	26	.31030	157	3.8089	31
13.20	1.9455	140	.50122	341	.44676	254	.27902	67	.30717	313	3.8029	60
13.40	1.9593	138	.49788	334	.44429	247	.27836	66	.30411	306	3.7967	62
13.60	1.9730	137	.49462	326	.44185	244	.27771	65	.30103	308	3.7905	62
13.80	1.9866	136	.49142	320	.43946	239	.27705	66	.29804	299	3.7844	61
14.00	2.0000	134	.48831	311	.43712	234	.27639	66	.29513	291	3.7783	61
14.20	2.0133	133	.48526	305	.43482	230	.27574	65	.29231	282	3.7721	62
14.40	2.0264	131	.48228	298	.43258	224	.27509	65	.28956	275	3.7659	62
14.40	2.0264	131	.48228	298	.43258	224	.27509	65	.28956	275	3.7659	62
14.60	2.0394	130	.47938	290	.43037	221	.27447	62	.28678	278	3.7598	61
14.80	2.0522	128	.47654	284	.42822	215	.27382	65	.28422	256	3.7536	62
15.00	2.0650	128	.47373	281	.42609	213	.27321	61	.28154	268	3.7475	61
15.20	2.0776	126	.47100	273	.42401	208	.27257	64	.27900	254	3.7413	62
15.40	2.0900	124	.46835	265	.42198	203	.27195	62	.27651	249	3.7351	62
15.60	2.1020	120	.46580	255	.42003	195	.27136	59	.27411	240	3.7290	61
15.80	2.1146	126	.46316	264	.41801	202	.27074	62	.27165	246	3.7228	62
16.00	2.1268	122	.46063	253	.41606	195	.27013	61	.26928	237	3.7167	61
16.20	2.1388	120	.45816	247	.41416	190	.26952	61	.26696	232	3.7104	63
16.40	2.1507	119	.45575	241	.41230	186	.26894	58	.26468	228	3.7045	59
16.60	2.1625	118	.45337	238	.41046	184	.26834	60	.26244	224	3.6986	58
16.80	2.1742	117	.45104	233	.40865	181	.26775	59	.26025	219	3.6928	57
17.00	2.1858	116	.44876	228	.40688	177	.26717	58	.25814	211	3.6871	56
17.20	2.1973	115	.44652	224	.40513	175	.26658	59	.25600	214	3.6815	57
17.40	2.2086	113	.44433	219	.40343	170	.26602	56	.25399	201	3.6758	55
17.60	2.2199	113	.44216	217	.40173	170	.26545	57	.25192	207	3.6703	55
17.80	2.2311	112	.44004	212	.40007	166	.26488	57	.24996	196	3.6648	52
18.00	2.2422	111	.43795	209	.39844	163	.26434	54	.24796	200	3.6596	52
18.20	2.2533	111	.43588	207	.39681	163	.26377	57	.24598	198	3.6538	58
18.40	2.2642	109	.43387	201	.39523	158	.26322	55	.24411	187	3.6483	55
18.60	2.2750	108	.43190	197	.39367	156	.26268	54	.24220	191	3.6427	56
18.80	2.2858	108	.42994	196	.39213	154	.26214	54	.24038	182	3.6372	55
19.00	2.2965	107	.42801	193	.39061	152	.26160	54	.23858	180	3.6317	55
19.20	2.3070	105	.42614	187	.38913	148	.26107	53	.23689	169	3.6261	56
19.40	2.3175	105	.42429	185	.38766	147	.26055	52	.23509	180	3.6206	55
19.60	2.3285	110	.42236	193	.38613	153	.26000	55	.23334	175	3.6152	54
19.80	2.3383	98	.42065	171	.38477	136	.25950	50	.23167	167	3.6096	56
20.00	2.3486	103	.41888	177	.38336	141	.25898	52	.23000	167	3.6040	56
21.00	2.3988	502	.41043	845	.37661	675	.25647	251	.22205	795	3.5777	263
22.00	2.4473	485	.40258	785	.37029	632	.25406	241	.21465	740	3.5536	241
23.00	2.4942	469	.39527	731	.36436	593	.25173	233	.20781	684	3.5286	250
24.00	2.5397	455	.38841	686	.35878	558	.24949	224	.20134	647	3.5058	228

probability paper, as noted under step 3 above, as a check on linearity. Forty C_s (or C_v) values proportionally spaced throughout Table 2 constituted the sample tested.

7. First differences, ΔK are shown in Table 2 as an aid to prospective users.

Needless to say, the five computed K-values for each entry of C_s will allow further probability levels to be obtained by the author's Fig. 1 method utilizing tabulated K-values for specified C_v values and solving the author's Eq. 1. (If publication space were available the 24 columns, 32 lines and 8 pages of tabulations of the individual steps could be made available). Table 2 does not contain an additional forty-eight computed C_s—K-values which have been computed to date for specific problems and to satisfy the curiosity of the writer. A brief summary of similar work by the writer as noted above is available.[9]

Precaution Necessary.—As pointed out in detail and in examples,[4] K-values may be used with the original variate or with a log-transformation variate. The latter is more efficient, statistically speaking, than the scalar transformation of the original variate. H. Alden Foster suggests[11] a method for making theoretical corrections to C_v and C_s. Similar work by the writer also supports this generalized approach.

[11] Discussion of "The Log-Probability Law and Its Engineering Applications," by H. Alden Foster, 1959, Proc. No. 709, Vol. 81, pp. 14-19.

ROLL WAVES AND SLUG FLOWS IN INCLINED OPEN CHANNELS[a]

Discussion by R. Hugh Taylor and John F. Kennedy

R. HUGH TAYLOR,[1] A.M. ASCE and JOHN F. KENNEDY,[2] A.M. ASCE.— The author has explored an obscure byway of hydraulics and has presented an interesting body of experimental data resulting from what was obviously a painstaking and thorough laboratory investigation. While knowledge is valuable per se, the fact that the phenomena dealt with here are so little known confers an additional value upon the results, they offer the engineer a means of broadening his intuition in an area he is not likely to have experienced directly. In this respect, then, the author has done the profession a definite service.

It is to be regretted, however, that the author's treatment of his results leads one to conclude that they are more directly useful than they actually are. This misunderstanding seems to have two different sources. First, the choice of nomenclature is unfortunate in that the term "roll waves" already has a generally accepted meaning, if not a precisely defined one. In spite of this, however, the author has chosen to apply the term exclusively to instabilities of laminar flow, a phenomenon which is of predominately academic interest to the hydraulic engineer. Furthermore, the term is so defined in the paper as to exclude the meanings already well established in the literature, notably in the writings of Thomas[4] and of Dressler.[5] Second, the confusion is enhanced by the reference to impact and freeboard problems in channel design caused by instabilities of flow. Certainly, the laminar instabilities the author calls "roll waves" are not of this magnitude, if they do not occur at Reynolds numbers greater than 415.

The author has likewise limited himself by choosing to include within his term "slug flow" only those unsteady flows which have arisen more or less directly from laminar flow. This may well be a reservation without a reason, if the properties of the limiting wave are not influenced by the presence or absence of prior laminar flow. Unless this is true, however, the similarity of these waves to the roll waves reported by Cornish[13] or those discussed by Dressler[5] is questionable. It seems improbable that the flow with roll waves which Dressler attributes to Cornish, or the flow with roll waves observed by V. A. Vanoni and the writers in the Santa Anita Flood Control Channel, near Arcadia, Calif., could ever have been laminar. This is not to object to the scope of the paper, but merely to question the relation between the actual scope and that which is suggested by the title and introduction.

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[a] July, 1959, by Paul G. Mayer.

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Purpose.—The purpose of this discussion is three-fold and may be suggested by a passage from the paper itself: "Rigorous mathematical treatment of complex hydraulic phenomena encounters great difficulties. Convenient approximations often lead to equations of questionable veracity. The background of experimental reality provides important insight into physical phenomena." Specifically, the writers' intent is to call attention to some theoretical arguments in a recent English paper, to enter an objection to what the author presents as the theory of "roll waves" and to the presentation and analysis of his experimental results, and to mention some recently reported experiments which supplement the findings of the author. The objections will be entered first.

On the Theory of "Roll Waves" and the Presentation and Analysis of Data.—The first objection is that the theoretical development is weak. If one assumes irrotational flow and surface disturbances small in amplitude with respect to their wave length, then the linear analysis of Airy[2] follows. It is well known that in this analysis the celerity, c, of a wave is given by

$$c^2 = \left(a\,\Gamma + \frac{g}{a}\right)\tanh\,(a\,D) \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(i)}$$

where $\Gamma = \frac{\sigma}{\rho}$ = "kinematic surface tension," so called by analogy with kinematic viscosity, and $a = \frac{2\pi}{\lambda}$ = wave number

Under the usual simplifications, tanh (a D) is taken equal to a D when a D is small, and it is taken as unity when a D is large (larger than $\lambda/2$, usually). In the former case one speaks of shallow water waves, the expression for the celerity of which has no useful minimum; that is,

$$c_{shallow} = \sqrt{\Gamma\, a^2\, D + g\, D}$$

from which

$$\frac{dc}{da} = \frac{\Gamma\, a\, D}{\sqrt{\Gamma\, a^2\, D + g\, D}}$$

The latter vanishes only for a = 0, which requires an infinite wave length or no disturbance at all. Of the two simplifications, it is only in the latter one, where one speaks of deep water waves, that one finds a minimum clerity and this case is certainly irrelevant to the problem at hand. The analysis in Eqs. (1) through (6) is for the deep water case, and since the author makes almost no use of his discussion of wave celerity, perhaps it would have been better to omit it completely.

The author's criterion for the formation of progressive waves is very hard to accept in the present situation. If the progressive waves associated with "roll waves" and "slug flow" are treated as discontinuous surge fronts, and the velocity is assumed constant with depth, the celerity of the waves can be shown to be

$$c = \sqrt{g\,D}\sqrt{\frac{1}{2}\frac{D}{D_o}\left(\frac{D}{D_o} + 1\right)} \quad \ldots\ldots\ldots\ldots \quad \text{(ii)}$$

where c is the velocity of the wave relative to the undisturbed flow, D represents the depth of undisturbed flow, and D_o is the depth upstream from the disturbance. If the disturbance is vanishingly small, that is, if $D = D_o$, then

the celerity becomes that given by the author in Eq. 11, $c = \sqrt{g\,D}$. The same limiting value is given by Eq. i for shallow water waves if surface tension is neglected and if the disturbance is very small compared to the wave length and the depth, that is, in the deep water case.

For Eq. ii to be valid, the velocity must not vary with depth, upstream and downstream from the disturbance. If some other velocity profile, such as the parabolic one used by the author, exists upstream and downstream from the disturbance, the wave celerity is further modified, as was shown by Boussinesq,[18] to

$$c = \sqrt{g\,D}\left(1 + \frac{3}{4}\frac{\eta}{D}\right)\left[1 + \frac{1-\beta}{2}\left(N_F^2 + 2\,N_F\right)\right] \quad \text{(iii)}$$

where η is the amplitude of the disturbance, N_F represents the Froude Number of the undisturbed flow, β is the momentum coefficient $= \frac{1}{D}\int_0^D (\nu/V)^2 dy$ and c denotes the velocity of the disturbance relative to the average velocity of the undisturbed flow.

The limiting case is again $c = \sqrt{g\,D}$, but only if $\eta = 0$ and $\beta = 1$ (a rectangular velocity profile). A comparison of Figs. 17 and 23 indicates that in the experiments reported by the author, the depth of flow and the wave heights were of the same order of magnitude. Noticing also that the Froude numbers were quite high, and recalling that the momentum coefficient has the value 1.20 for a parabolic velocity distribution, one must conclude that the use of $\sqrt{g\,D}$ as the wave velocity relative to the average velocity of flow is incorrect.

Actually, taking the Froude number as 1.7, the amplitude of the disturbance as half the undisturbed depth, and a parabolic velocity distribution, the resulting disturbance velocity is only about half that given by the limiting case. Similarly, the celerity of oscillatory waves (Eq. i) is greatly modified if the velocity profile departs appreciably from rectangular. Thus Eq. 11 and the criteria stated in Eqs. 12a and 17 must be rejected.

The kinematic argument of Lighthill and Whitham[3] properly predicts a celerity of very long, "kinematic" waves (in which dynamic considerations are unimportant) equal to twice the mean stream velocity. But to convert this to the statement that the Froude number equals 2 is equivalent to stating that the wave celerity is given by $\sqrt{g\,D}$, which was shown above to be incorrect for the problem at hand. Lighthill and Whitham themselves said: "It must be remarked that the result $N_F < 2$ has been deduced under the assumption that the disturbance is small. More generally, . . . , the value of N_F which must be exceeded if a bore of constant strength is to be maintained, depends on the strength; this value of N_F is always less than 2 and tends to 2 as the strength approaches zero."

The second objection to be entered is that the author's analysis of the experimental data is somewhat misleading. Where dimensionless parameters are used, it is generally desirable to use the standard ones for which the reader already has an intuitive "feel," and to resist the strong temptation to invent new ones unless the gain of greater simplicity of the new ones outweighs the advantages of familiarity with the established ones. The parameter presented in Eq. 29 which he calls the critical flow number F_{cr}, is actually a modified Weber number:

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18 "Essai sur la Théorie des Eaux Courantes," Mémoires Divers Savants à l'Académie des Sciences, 1877, Vol. 23.

$$F_{cr} = \left(\frac{\rho q \nu}{\sigma S^{1/6}}\right)^{\frac{3}{2}} = \left(\frac{N_W}{S^{1/6}}\right)^{\frac{3}{2}} \quad \text{(iv)}$$

This results from the tacit assumption that the Froude number is unity in the development of the critical flow number in Eq. 29.

The critical depth number, N_D, which is given in Eq. 31, can also be considered as a shear velocity Reynolds number,

$$N_D = \left(\frac{g D^3 S}{\nu^2}\right)^{\frac{1}{2}} = \frac{u_* D}{\nu} \quad \text{(v)}$$

where u_* = shear velocity = $\sqrt{g DS}$. The Darcy-Weisbach friction factor, f, is defined by

$$S = \frac{f}{4 D} \frac{V^2}{2 g} \quad \text{(vi)}$$

and can be expressed as

$$f = 8\left(\frac{u_*}{V}\right)^2 \quad \text{(vii)}$$

or

$$u_* = V\sqrt{f/8}$$

from which

$$N_D = R_D\sqrt{f/8} \quad \text{(viii)}$$

In the laminar regime, where a parabolic velocity profile is applicable, Eqs. 10a and (vi) yield

$$f = 24/R_D \quad \text{(ix)}$$

Substituting this value of f in Eq. viii gives

$$N_D = \sqrt{3 R_D} \quad \text{(x)}$$

In the present experiments, a single fluid was used and thus the only variables in the critical flow number are V, D, and S. Thus Fig. 16, in which N_D is plotted against F_{cr}, essentially relates V, S, and D, and would be expected to give a linear relation in the laminar region on a logarithmic plot. With a suitable manipulation of variables, Fig. 16 can be put in the form of the usual pipe friction diagram (except for the constants involving ρ and σ) which is found in any elementary text on fluid mechanics.[19] For flow of this type, the laminar region ends at approximately R_D = 600 to 1,000 (although laminar flow can exist at Reynolds numbers up to 2,500 under appropriate conditions) which corresponds to N_D = 42.5 to 54.8. In Fig. 16, this is about the region in which the straight line relation ends, as would be expected. It is also the upper limit of the Reynolds number for which roll waves were observed and this confirms the conclusion that "roll waves" are confined to the laminar regime of flow.

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19 "Fluid Mechanics," by R. L. Daugherty, and A. C. Ingersoll, McGraw-Hill, 1955 p. 182.

In a similar manner, Figs. 12, 13, 14, 15, and 18 are also essentially pipe-friction diagrams for this type of flow. This becomes more apparent when it is considered that

$$\frac{N_F}{\sqrt{S}} = \left(\frac{8}{f}\right)^{\frac{1}{2}}$$

Indeed, if use is made of Eq. ix the result for the laminar regime is

$$\frac{N_F}{\sqrt{S}} = \left(\frac{8\ R_D}{24}\right)^{\frac{1}{2}}$$

or

$$N_F = \sqrt{S/3}\ \sqrt{R_D}$$

which is exactly the relation the author presents as Eq. 19 and in Fig. 18. The runs conducted outside of the laminar regime are in the transition regime from laminar to turbulent flow, thus the friction factor cannot be obtained directly from a pipe friction diagram, since the relation between R_D and f in this region is not well defined. Actually, it depends on the manner in which the transition occurs, which in turn depends on the manner in which the experiments were conducted. Had the experiments progressed into the fully turbulent regime, the relation between S, V, and D could be obtained directly from a pipe friction diagram. Thus it appears that this phase of the author's analysis consists primarily of presenting the well known pipe friction diagram in several alternate forms.

The author's statement that the minimum slope for "roll wave" formation is 3°/o was not actually verified by his experiments since no runs were made with slopes in the neighborhood of 3°/o. Actually Figs. 16, 17, and 18 would indicate that a better criterion for the formation of "roll waves" and slug flow as defined by the author would be that "roll waves" are associated with laminar flow, while slug flow occurs when the flow is in the transitional regime between laminar and turbulent flow. The writers also question the applicability of the statement: "All Froude numbers leading to the formation of slug flows were above the theoretical minimum $N_F = 2$". The criterion for slug flow formation mentioned above would indicate that slug flow could result even if $N_F < 2$ if the flow were transitional or turbulent. No experiment was conducted in this range to either prove or disprove the necessity of turbulence for slug flow formation regardless of the size of the Froude number. Such an experiment would be very enlightening.

Comparison with Other Recent Papers.—There are two papers (by A. M. Binnie[20] and T. B. Benjamin[21]) which appeared at about the same time* as

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20 "Instability in a Slightly Inclined Water Channel," by A. M. Binnie, J. Fluid Mech., 1959, vol. 5, pp. 561–570.

21 "Wave Formation in Laminar Flow Down an Inclined Plane," J. Fluid Mech., by T. B. Benjamin, 1957, vol. 2, pp. 554–574; Numerical corrections ibid. 1958, vol. 3, p. 657.

* The actual precedence is somewhat confusing. The theoretical paper of Benjamin was published before the present paper (but after the thesis upon which it is based). Similarly, the experimental results of Binnie appeared after the oral presentation of the present paper but before it was published. As an aside, it might be suggested that the date of submission of a paper be shown in case questions of priority ever should arise. This is common procedure in many technical journals.

the present one, and which are relevant to this discussion. They are mentioned at some length here because they appeared in a relatively new publication, the Journal of Fluid Mechanics, which is published in England, and may not be familiar to some readers. The two papers are essentially contemporary with the author's; it is interesting to see if they are in agreement.

The English papers supplement the present one in that (a) they are concerned with the lower threshold of waves in laminar flow, (b) Binnie reports having explored the spectrum of Reynolds numbers from before occurrence of waves up into the turbulent patch or "slug flow" condition, and (c) Benjamin has developed a simplified stability analysis, not dependent upon the Airy assumptions, which Binnie has applied to laminar flow in slightly inclined open channels.

A brief review of Benjamin's stability analysis is presented by Binnie, as adapted for small slopes. It is essentially as follows: Define the following dimensionless quantities:

Celerity	$C = c/V = C_r + i\,C_i$, C_r and C_i both being real
Wave number	$\alpha = a\,D$
Distance	$X = x/D$, $Y = y/D$
Time	$T = t\,D/V$.

Now if one imposes a small disturbance $Y = \delta \exp\{\alpha\, i(X - C\,T)\}$ on the laminar flow represented by Eqs. 7 through 10, there results an equation known as the Orr-Somerfeld equation.[22] This equation can be solved in power series in R_D which converges fairly rapidly for small R_D, although the algebra is quite tedious. For very small waves one finds $C_r = 2$ (thus confirming Lighthill and Whitham under the given assumptions), and

$$C_i = \frac{1}{2} R_D \left(\frac{8}{5}\alpha - \frac{\alpha^3 \Gamma}{D\,V^2} - \frac{\alpha}{V^2} g\,D \cos\theta\right) \qquad \text{(xi)}$$

where θ is the inclination of the channel. Within the small-wave assumption, all this equation can show is the relation between R_D, D, and θ under which the waves would start to grow exponentially. The condition for stability is that $G_i < 0$, so that the disturbance will have no part which grows exponentially with time. Thus, for instability, we must have

$$\frac{\alpha^2 \Gamma}{D\,V^2} + \frac{D}{V^2} g \cos\theta < \frac{8}{5} \qquad \text{(xii)}$$

Of interest is the wave number, α_m, for the fastest growing instability. This is obtained by maximizing $\alpha\,C_i$ with respect to α and is given by

$$\alpha_m^2 = \frac{D\,V^2}{\Gamma}\left(\frac{4}{5} - \frac{D}{2\,V^2} g \cos\theta\right) \qquad \text{(xiii)}$$

If this last result, together with the primary flow Eqs. 8 and 10 be substituted into Eq. xiii, the maximum value of C_i is seen to be

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22 "The Theory of Hydrodynamic Stability," by C. C. Lin, Cambridge Univ. Press, 1955.

$$\max C_i = 0.625\left(\frac{4}{5} - \frac{2}{3}\frac{\cot\theta}{R_D}\right)^{3/2} \frac{\nu^{2/3}}{\Gamma^{1/2}} (g \sin\theta)^{1/6} R_D^{11/6} \quad . . \text{(xiv)}$$

It is seen that the minimum R_D for which instability can exist is obtained by setting $\max C_i = 0$. This gives

$$R_D = \frac{5}{6} \cot\theta \quad \ldots\ldots\ldots\ldots\ldots \text{(xv)}$$

Because of the linearizations involved in its derivation, the development mentioned above cannot lead to definite conclusions concerning fully-developed

TABLE 1.—THRESHOLD OF INSTABILITY. COMPARISON OF CALCULATED WITH OBSERVED RESULTS, IN TERMS OF DEPTH REYNOLDS NUMBERS

θ (degrees)	R_D'	R_D''
1	48	56
2	24	37
2 3/4	17	29
3	16	. .
4	12	. .
5	9.6	. .

Note: R_D' is given by $R_D' = \frac{5}{6} \cot\theta$, Eq. xv. R_D'' is the threshold reported by Binnie.

"roll waves." However, it can indicate a method of attack, and it can also be used to give a lower bound to the values of R_D below which unstable waves, and the "roll waves" arising from them, cannot occur. For this purpose the Reynolds numbers corresponding to neutral stability (that is, to $C_i = 0$) were calculated, and the values superimposed on the author's Fig. 17. Binnie's observed thresholds are also shown, for $\theta = 1^\circ$, 2°, and $2\ 3/4^\circ$. (See Fig. 27.) These values are well below the author's observed values and therefore can be said to be consistent.

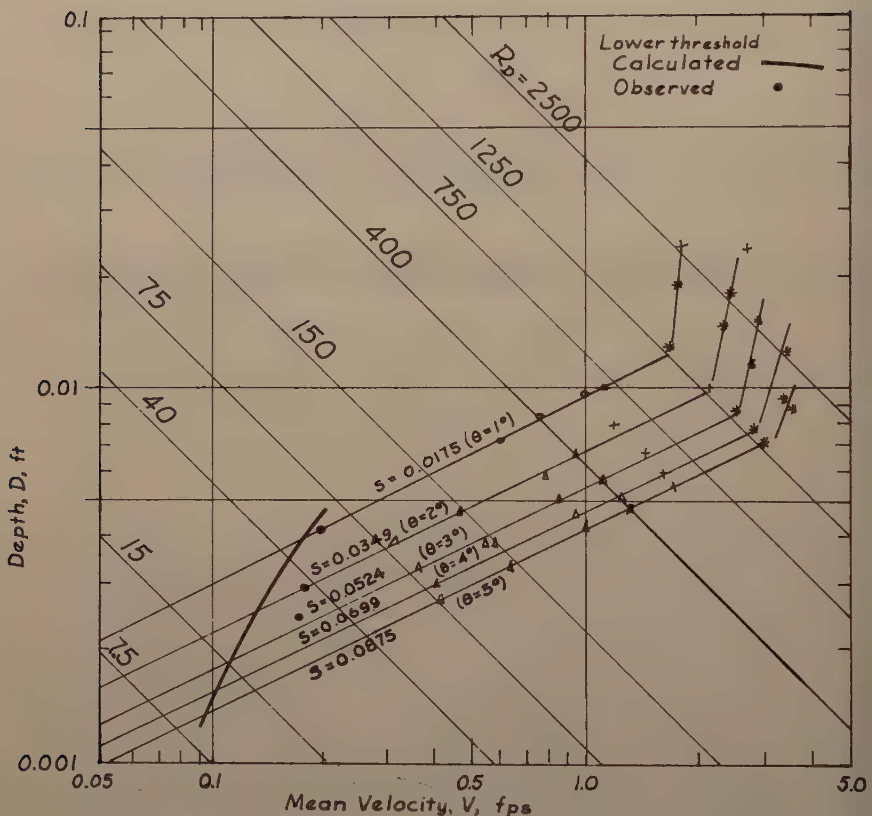

FIG. 27.—COMPARISON OF MAYER'S RESULTS WITH LOWER THRESHOLDS REPORTED BY BINNIE (CF. FIG. 17).

FLOOD CONTROL ASPECTS OF CAUCA VALLEY DEVELOPMENT[a]

Discussion by Gordon R. Williams

GORDON R. WILLIAMS,[1] F. ASCE.—The planning and final design of water resources developments should be preceded by a thorough study of the basic meteorologic and climatologic characteristics of the general region. Such a study should first lead to a classification of the climate according to type along the lines suggested by Köppen, Thornwaite and others. It is evident that such a study was made for the Cauca Valley, and although not specifically mentioned, the climate falls into Köppen's "tropical wet and dry" or "savanna" classification. Such climates extend along the entire western coast of Central America and into Mexico.

The purpose in classifying the climate as to type is to determine in a general way the potentialities of the climate, including the possible occurrence of different storm types, the probable severity and duration of droughts, and the possibility of killing frosts and damaging winds. For example, the tropical wet and dry climate can be expected to have a total annual precipitation which will be enough to supply the annual evapo-transpiration to be expected from consideration of the regional temperature and humidity, but the distribution of the precipitation month by month will vary widely and there will be seasonal deficiencies which will result in crop failures without irrigation. The Cauca Valley definitely has this type of climate and, therefore, irrigation works are necessary to develop the agricultural potential of the region.

The paper reveals that the storm rainfall potentials in the region are low, relative to many other parts of the world, because of the absence of hurricanes or cylconic disturbances associated with cold fronts. As a measure of this characteristic, it is pointed out that the maximum 24-hr rainfall has never exceeded 6% of the average annual rainfall. This is in contrast to the east coast of the United States where 24-hr storm rainfalls equal to 15% to 20% of the average annual rainfall have occurred over areas of several thousand square miles. Presumably most of the precipitation in the Cauca Valley results from convective cells, which are characteristics of the doldrum region. The peak of this convective activity occurs in Colombia, in the northern hemisphere's winter, spring or late fall seasons, when the solar energy in the vicinity of the equator is greatest. The seasonal rainfall patterns result from an alternate north and south migration of the centers of convective action in unison with the causative solar influences. No mention is made of monsoon conditions, but undoubtedly the mountain barriers to the west would prevent the development of any strong moisture-bearing monsoon winds from the Pacific Ocean.

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a September, 1959, by Phillip Z. Kirpich and Carlos S. Ospina.

1 Prof. of Hydr. Engrg., Mass. Inst. of Tech., Cambridge, Mass.

The method of determining the spillway design storm and flood is unusual and is dictated by a lack of data and by the meteorologic characteristics of the storm rainfall. The method has considerable justification but some of the details of the analysis might be questioned. The attempt to evaluate the 1,000-yr frequency for the 5-day precipitation from the 18-yr record is mathematically unrealistic. The rainfall record is so short that the slope of the frequency curve is very uncertain and values taken from a long extrapolation may not necessarily be conservative. In spillway design storms the object is to be conservative and to resort to the factor of safety principle may be the only rational approach. The method of expressing the rainfall depths as a percentage of the annual rainfall could be adopted for all durations from 1 to 5 days. Then comparisons could be made with corresponding percentages that have been experienced in similar climates in other parts of the world, where the rainfall records are much longer. If the climate of a river basin has been correctly classified, the writer believes that statistical analogies may be made even from one continent to another.

The procedure of plotting the known 5-day rainfall totals against the maximum daily flow is practical and expedient in view of the lack of records and the non uniform character of the rainfall. However, the position of the envelope curve is subject to individual judgment. It is questionable whether a straight line on semi-log paper is proper. If the plot is transferred to an arithmetic scale, the envelope curve will be found to be concave upward in the general form of a parabola with a vertical axis. If the infiltration losses for the storms are essentially constant, the envelope curve certainly violates the unity-hydrograph theory which shows that hydrograph peaks vary about as the first power of the volume of rainfall excess. If natural storage, of either channel, valley or lake type, is present there will be a tendency for the envelope curve to be concave downward on an arithmetic scale. On the basis of the above discussion, and assuming that 186 mm represents the correct factor of safety for the rainfall it is difficult to justify a daily flow in excess of 2,500 cu m per sec. The computed peak discharge of 6,000 cu m per sec with a Myers rating of 4,550, appears to place the river in a climatic province in which it obviously is not. As one schooled in the United States method of determining the maximum possible flood for spillways for major dams, the writer would be the last one to deplore the use of factors of safety in spillway design, but, in this case, it appears that the planners of the project do not have faith in their own very exhaustive meteorologic and hydrologic appraisal of the region.

The adopted plan for flood protection and drainage of the Aguablanca Project, benefits from the application of hydrologic and hydraulic ingeniuity as well as from the sound economic analysis. The intercepting canal on the west, which carries the three mountain streams around the protected area, is a good solution which is undoubtedly aided by favorable land slopes in a southerly direction. The use of an excavated pond to reduce pumping capacities is a good practice which has been applied to the drainage of leveed areas in the United States, and, in some cases, to reduce the size of storm sewers in ordinary urban drainage problems.

FRICTION FACTORS IN CORRUGATED METAL PIPES[a]

Discussion by M. H. Diskin

M. H. DISKIN.[1]—This paper is a valuable contribution to the knowledge of pipe flow in providing accurate experimental data on loss of head in corrugated pipes of large size. It is to be regretted that the experiments could not be extended to cover a larger range of Reynolds numbers, so as to show the trend of the f vs. Re curves before and after the maximum point observed, and in particular to show if the friction coefficient becomes constant at higher Reynolds numbers.

With reference to Fig. 17 of the original paper, it is interesting to note that, if the recommended values of friction coefficient (corresponding to the maximum observed values) are considered as representing the values for rough turbulent flow, the equivalent pipe roughness, based on these values, is nearly the same for the three pipes and agreés with that computed for the smaller pipes reported. Taking Nikuradse's equation:

$$1/\sqrt{f} = 1.74 + 2 \log (D/2\epsilon)$$

for the definition of the equivalent roughness (ϵ), the values computed for the various pipes reported are given in Table 1.

TABLE 1

Nominal diameter, in feet	D, in feet	f	$1/\sqrt{f}$	(D/ϵ)	ϵ, in feet
7	7.05	0.054	4.303	38.38	0.18
5	4.95	0.064	3.953	25.56	0.19
3	3.00	0.077	3.604	17.09	0.18
2	---	0.092	3.297	12.01	0.17
1.5	---	0.102	3.131	9.92	0.15

a September, 1959, by Marvin J. Webster and Laurence R. Metcalf.
1 Hydr. Lab., Technion, Haifa, Israel.

REVISED COMPUTATION OF A VELOCITY[a] HEAD WEIGHTED VALUE

Discussion by Manuel A. Benson

MANUEL A. BENSON,[1] M. ASCE.—The revised computational procedure proposed by the authors is apparently motivated by the difficulty of handling the extremely large numbers encountered in computing the velocity-head coefficient α. This difficulty is easily resolved by a coding process.

Table 1 shows the slide-rule computation of α for a cross-section which has been subdivided into five parts on the basis of hydraulic radius and roughness. Cols. 2 and 3 show the areas (a) and conveyances (k) of the separate subsections.

Lines are drawn arbitrarily through the a and k columns, in effect shifting the decimal point, so that k/a-values are, in general, small numbers greater than 1.

Col. 4 shows the slide-rule computations of k^3/a^2. All the significant figures in Cols. 2 and 3 are used for this computation. Cols. 5, 6, and 7 are rapid mental computations based on only the figures to the left of the lines in the a and k columns. These computations are made in order to locate the decimal points in the slide-rule computations of Col. 4. The correctly positioned figures are shown in Col. 8. Cols. 4 through 7 need not be recorded; they are shown only to illustrate the mental processes involved in locating the decimal points.

The values in Col. 8 are coded values and do not represent the actual values of the cubed conveyance divided by the squared area. However, α, computed as

$$\frac{\Sigma\,(k^3/a^2)}{K^3/A^2}$$

is dimensionless and is correct regardless of the coding used.

If the geometric mean of the conveyances at both ends of a reach is used in the discharge formula $Q = K_m\, S^{\frac{1}{2}}$, it is possible to develop a formula for Q which makes unnecessary a trial-and-error procedure such as the authors use. The formula is:

a September, 1959, by J. M. Lara and K. B. Schroeder.

1 Hydr. Engr., Research, U. S. Geol. Survey, Washington 25, D. C.

$$Q = K_2 \sqrt{\frac{\Delta h}{\left(\frac{K_2}{K_1}\right) L_{1.2} + \left(\frac{K_2}{A_2}\right)^2 \frac{E_{1.2}}{2\,g} \left[-\,\alpha_1 \left(\frac{A_2}{A_1}\right)^2 + \alpha_2\right]}}$$

in which Q is the discharge in cubic feet per second; K_1 and K_2 are the tota conveyances in section 1 (upstream) and section 2 (downstream) respectively-

$$K = \frac{1.486}{n} A\ R^{2/3}$$

$L_{1.2}$ is the length of reach, in feet, between sections 1 and 2; Δh denotes th water-surface drop, in feet, between sections 1 and 2; g is the constant o gravity, 32.2 fps^2; α_1 and α_2 represent the velocity-head coefficients a sections 1 and 2; and $E_{1.2}$ is the coefficient representing the percentage o

TABLE 1

Subsection	Area a	Conveyance b	Slide rule value of k^3/a^2	-----Approximate values of----- k/a	$(k/a)^2$	$(k/a)^2 k$	k^3/a^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	2,515	331,000	573	1	1	33	57.3
2	1,263	298,000	1660	2	4	120	166.0
3	313	63,000	268	2	4	24	26.8
4	1,142	242,000	1085	2	4	96	108.5
5	1,612	160,000	1570	1	1	16	15.7
Total	6,845(A)	1094,000(K)	281	2	4	400	281.0

$$\alpha = \frac{\Sigma(k^3/a^2)}{K^3/A^2} = \frac{374.3}{281.0} = 1.33$$

recovery of velocity head. For expanding reaches, $E_{1.2}$ is commonly as- sumed as 0.5; for contracting reaches, $E_{1.2}$ is assumed as 1.0 or 0.9. A reach is contracting if the bracketed quantity in the denominator is positive expanding if it is negative.

This method, like that of the author's, permits computation of velocitie and discharges in each subdivision. It is necessary only to proportion th total discharge on the basis of the relative conveyance in each subdivision.

Using the base data of the author's sample computations (areas, convey- ances, and length of reach), the writer took 17 min to compute the two value of α and a discharge of 10,700 cfs by means of the suggested formula for di- rect computation of discharge.

The U.S. Geological Survey has developed similar formulas for multiple- section reaches covering rather complex conditions, such as alternate con- traction and expansion or abrupt contractions at a bridge opening. These formulas provide direct solutions and eliminate trial-and-error procedures.

REVISED COMPUTATION OF A VELOCITY HEAD WEIGHTED VALUE[a]

Discussion by Marcel Bitoun

MARCEL BITOUN,[1] M. ASCE.—Just to keep the record straight, Bernouilli's theorem, in the Appendix to Messrs. Lara & Schroeder's paper, is not correctly applied. The eddy loss h_t is always a subtractive term:

$$h_t = m \mid h_{v_1} - h_{v_2} \mid$$

where the signs $\mid \; \mid$ indicate that only the absolute value of the difference $h_{v_1} - h_{v_2}$ must be considered, irrespective of its algebraic sign.

Then, for a contracting reach, and if $m = 0.10$:

$$h_f = f + (h_{v_1} - h_{v_2}) - 0.10 \mid h_{v_1} - h_{v_2} \mid$$

or, since $h_{v_2} > h_{v_1}$:

$$h_f = f + (h_{v_1} - h_{v_2}) + 0.10 \; (h_{v_1} - h_{v_2})$$

$$= f + 1.10 \; (h_{v_1} - h_{v_2}).$$

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[a] September, 1959, by J. M. Lara and K. B. Schroeder.

[1] Chf. Design Branch, Div. of Flood Control, Pa. Dept. of Forests & Waters, Harrisburg, Pa.

DISCHARGE FORMULA FOR STRAIGHT ALLUVIAL CHANNELS[a]

Discussion by T. Blench

T. BLENCH,[1] F. ASCE.—The writer is interested in this paper as an example of the extreme swing away from formulas based mainly on theoretical speculations and idealizations, to empirical fitting curves that, apart from avoiding dimensional absurdity, seem to be unrelated to any dynamical considerations; however, they do seem to be biased in favor of the belief that Manning's formula is of the proper type, for most alluvial channels with dunes. A neat fit has been obtained to a miscellany of slightly incongruous data by the patient selection of sufficient indices of non-dimensional groups of variables; the various graphs, combined with the references to sources of data, should be most useful to laboratory research workers.

However, for various reasons, the writer would not use the various curves in practice. First, the data are mainly from laboratory flumes in which $V\,b/\nu$ lies between about 10^5 and 10^6 and the breadth-to-depth ratio, b/d, is probably round 3.0 or less; in the field $V\,b/\nu$ usually lies between 10^6 and 10^8 with b/d from about 5 to 30. Some of the data (Gilbert, Figs. 3,4) are for unnaturally uniformized bed material; some are for bed-conditions that had not become steady (Ref. 29); some are for mixed sediment with indeterminate but probably significant amounts in suspension (Refs. 23,29). The writer has already pointed out[2] that extrapolation of such data leads to results differing greatly from field experience in certain respects. Second, the curves and indices seem to have been adjusted to favor Manning's formula whereas[3,4] there is a large body of field evidence that Manning's index of 2/3 should be replaced by 3/4 to overcome the difficulty that his n is a function of hydraulic radius; the writer has experience of a canal system where, for apparently identical bed, n had to be specified at values from 0.018 for the main channels (10,000 cfs) up to 0.0225 for the small laterals (10 cfs)—these values, based on experience, correspond very closely to the indicial adjustment just mentioned. Third, no cognizance is taken of bed-load charge which is important in the Gilbert experiments, in many river problems, and in occasional special canal ones; for example, one test problem[4] showed that n altered from about 0.021 down to 0.017 for a given channel breadth and 0.25 mm sand as a consequence of changing bed-load charge from about zero to 0.04% by weight, and increasing the slope to permit carriage of the load. (Note that Manning's n may keep fairly constant for various discharges at one site in one channel

[a] November, 1959, by Liu and Hwang.

[1] Prof. of Civ. Engrg., Univ. of Alberta, and Cons. Engr.

[2] "Graphic Design of Alluvial Channels," by Ning Chien, Proc. ASCE Separate 611, Discussion by T. Blench.

because the effects of varying charge and side shear at varying stages counteract each other to some extent and the incorrect index, 2/3, acts to reduce the variation of n; these are not the conditions of the problem just mentioned.) Fourth, there are simple and dynamically satisfactory formulas for mobile-bed channels,[4] based on an enormous collection of field data, that solve all the practical problems of the present paper, plus others involving bed-load charge.

Although the so-called regime formulas[4] have the practical advantages of simplicity and as good accuracy as any in the field—since they were discovered by analyzing major field data—they do also have the fundamental advantage of being consistent with the friction-factor diagram of rigid-boundary hydraulics, which they generalize. In fact, the regime flow formula can be written for study (it is written differently and very simply for design purposes) as:

$$V^2/g\,d\,S = 3.63(1+a\,C)\,(V\,b/\nu)^{\frac{1}{4}} \qquad (1)$$

which is a generalized form of Blasius' equation, as it should be since the boundary is formed from the water-sediment complex just as the so-called "smooth" rigid pipe boundary is formed from the pure water. (C is bed-load charge and a is a constant.) Moreover, when we consider that a bed-factor can be defined as $F_b = V^2/d$ and a side-factor as $F_s = V^3/b$, the above equation can be converted into:

$$V = 3.63\,(d/x)^{\frac{1}{4}}\,\sqrt{g\,d\,S} \qquad (2)$$

in which x measures an equivalent roughness height and is equal to $(\nu F_s)^{\frac{1}{2}}/[F_b(1+a\,C)^2]$. So now the formula is of "rough boundary" form as it should be since the bed is covered with dunes. In fact, regime channels are rough and smooth at the same time, drawing attention to the artificiality of these adjectives, as used in hydraulics, and to the fact that there should be a universal flow formula which breaks into special forms for special boundary phases merely by expressing the equivalent roughness height properly; this is consistent with the belief in a universal velocity distribution, independent of boundary phase, as held by authorities on rigid-boundary hydraulics. Reverting to rigid-boundary speculations, one of them converts Eq. 2 into the form of the Blasius equation by replacing x by laminar film thickness.[5] If a universal flow formula exists, and if Eq. 2 is correct, then Manning for rigid boundary conditions must be amended to have 3/4 as an index instead of 2/3.

Statistically the writer would like to see confidence bands on all charts to draw attention to the amount of variation that might be reasonably expected. Although this necessary statistical refinement is essential for a proper understanding—its omission is comparable with describing the rainfall of a place by its long-term average without any indication of the scatter about the average—it is seldom used; a few hydrologists have begun to expound its value in order to prevent the usual misinterpretation of the significance of flood frequency curves. In the absence of confidence bands the writer can only give his opinion, based on experience of plotting hydraulic data, that quite a different looking set of indices would produce just as good fitting curves and, in particular, so would a set based on the belief that Manning's 2/3 ought to be replaced by 3/4.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division ponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air ransport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hy-raulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil echanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors VW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the sym-ols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning ith Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To cate papers in the Journals, the symbols after the paper number are followed by a numeral designating e issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural ivision during 1959.

VOLUME 85 (1959)

EBRUARY: 1933(HY2), 1934(HY2), 1935(HY2), 1936(SM1), 1937(SM1), 1938(ST2), 1939(ST2), 1940(ST2), 1941(ST2), 1942(ST2), 1943(ST2), 1944(ST2), 1945(HY2), 1946(PO1), 1947(PO1), 1948(PO1), 1949(PO1), 1950(HY2)[c], 1951(SM1)[c], 1952(ST2)[c], 1953(PO1)[c], 1954(CO1), 1955(CO1), 1956(CO1), 1957(CO1), 1958(CO1), 1959(CO1).

ARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986 (IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

PRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998 (SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007 (HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

AY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c], 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

JNE: 2048(CP1), 2049(CP1), 2050(CP1), 2051(CP1), 2052(CP1), 2053(CP1), 2054(CP1), 2055(CP1), 2056 (HY6), 2057(HY6), 2058(HY6), 2059(IR2), 2060(IR2), 2061(PO3), 2062(SM3), 2063(SM3), 2064(SM3), 2065 (ST6), 2066(WW2), 2067(WW2), 2068(WW2), 2069(WW2), 2070(WW2), 2071(WW2), 2072(CP1)[c], 2073(IR2)[c], 2074(PO3)[c], 2075(ST6)[c], 2076(HY6)[c], 2077(SM3)[c], 2078(WW2)[c].

ULY: 2079(HY7), 2080(HY7), 2081(HY7), 2082(HY7), 2083(HY7), 2084(HY7), 2085(HY7), 2086(SA4), 2087 (SA4), 2088(SA4), 2089(SA4), 2090(SA4), 2091(EM3), 2092(EM3), 2093(EM3), 2094(EM3), 2095(EM3), 2096 (EM3), 2097(HY7)[c], 2098(SA4)[c], 2099(EM3)[c], 2100(AT3), 2101(AT3), 2102(AT3), 2103(AT3), 2104(AT3), 2105(AT3), 2106(AT3), 2107(AT3), 2108(AT3), 2109(AT3), 2110(AT3), 2111(AT3), 2112(AT3), 2113(AT3), 2114(AT3), 2115(AT3), 2116(AT3), 2117(AT3), 2118(AT3), 2119(AT3), 2120(AT3), 2121(AT3), 2122(AT3), 2123(AT3), 2124(AT3), 2125(AT3).

UGUST: 2126(HY8), 2127(HY8), 2128(HY8), 2129(HY8), 2130(PO4), 2131(PO4), 2132(PO4), 2133(PO4), 2134 (SM4), 2135(SM4), 2136(SM4), 2137(SM4), 2138(HY8)[c], 2139(PO4)[c] 2140(SM4)[c].

EPTEMBER: 2141(CO2), 2142(CO2), 2143(CO2), 2144(HW3), 2145(HW3), 2146(HW3), 2147(HY9), 2148(HY9), 2149(HY9), 2150(HY9), 2151(IR3), 2152(ST7)[c], 2153(IR3), 2154(IR3), 2155(IR3), 2156(IR3), 2157(IR3), 2158 (IR3), 2159(IR3), 2160(IR3), 2161(SA5), 2162(SA5), 2163(ST7), 2164(ST7), 2165(SU1), 2166(SU1), 2167(WW3), 2168(WW3), 2169(WW3), 2170(WW3), 2171(WW3), 2172(WW3), 2173(WW3), 2174(WW3), 2175(WW3), 2176 (WW3), 2177(WW3), 2178(CO2)[c], 2179(IR3)[c], 2180(HW3)[c], 2181(SA5)[c], 2182(HY9)[c], 2183(SU1)[c], 2184 (WW3)[c], 2185(PP2)[c], 2186(ST7)[c], 2187(PP2), 2188(PP2).

CTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205 (PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214 (SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223 (ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)[c], 2233(PL3)[c], 2234(EM4)[c], 2235(HY10)[c], 2236(SM5)[c], 2237(ST8)[c], 2238(PO5)[c], 2239(ST8), 2240 (PL3).

OVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

ECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

VOLUME 86 (1960)

ANUARY: 2331(EM1), 2332(EM1), 2333(EM1), 2334(EM1), 2335(HY1), 2336(HY1), 2337(EM1), 2338(EM1), 2339(HY1), 2340(HY1), 2341(SA1), 2342(EM1), 2343(SA1), 2344(ST1), 2345(ST1), 2346(ST1), 2347(ST1), 2348(EM1)[c], 2349(HY1)[c], 2350(ST1), 2351(ST1), 2352(SA1)[c], 2353(ST1)[c], 2354(ST1).

EBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

[c] Discussion of several papers, grouped by divisions.

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